Supply Shocks and Household Heterogeneity in Open Economies: Implications for Optimal Monetary Policy*

Nicolai Waldstrøm†

September 2024

Abstract

I study the transmission of cost-push shocks in a small open economy using a Heterogeneous Agent New Keynesian (HANK) model. Compared to the canonical Representative Agent New Keynesian (RANK) model, I show that a HANK model with empirically realistic marginal propensities to consume out of income (MPCs) and sticky wages introduces an additional transmission channel: An increase in inflation following a cost-push shock suppresses real wages, which suppress aggregate demand when the MPC out of labor income is greater than the MPC out of profits, highlighting the distributional role of inflation. I then compute the optimal monetary policy response to an increase in import prices. I find that a more hawkish response is optimal in HANK compared to RANK. This is driven by low short-run trade elasticities combined with positive exchange rate pass-through to import prices, implying that an exchange rate appreciation can stabilize inflation and real wages without significantly lowering domestic employment.

*I am indebted to my supervisors Jeppe Druedahl and Søren Hove Ravn for their continuous support. I am thankful to Christian Bayer, Luigi Bocola, Tobias Broer, John V. Kramer, Patrick Moran, Morten Ravn, Pontus Rendahl, Federica Romei, Jacob Marott Sundram, Felix Wellschmied and seminar participants at NORMAC 2024, the University of Copenhagen and DGPE 2022 for helpful comments. Financial support from the Carlsberg Foundation (Grant CF20-0546) is gratefully acknowledged.

†Department of Economics, University of Copenhagen. Email: nicolai.w@econ.ku.dk

1 Introduction

Recently many advanced economies have experienced large surges in inflation brought about by higher import prices (see left panel in Figure 1). Rigid nominal wages imply that households have experienced a decline in real wages and income (see right panel in Figure 1), leading some economists to dub the recent situation as a ”cost-of-living crisis”.[1] This reduction in real income imply that inflation may have a direct effect on aggregate demand and on welfare. In this paper I study the transmission of inflation through real income on aggregate demand, and evaluate the implications for optimal monetary policy in a small open economy (SOE).

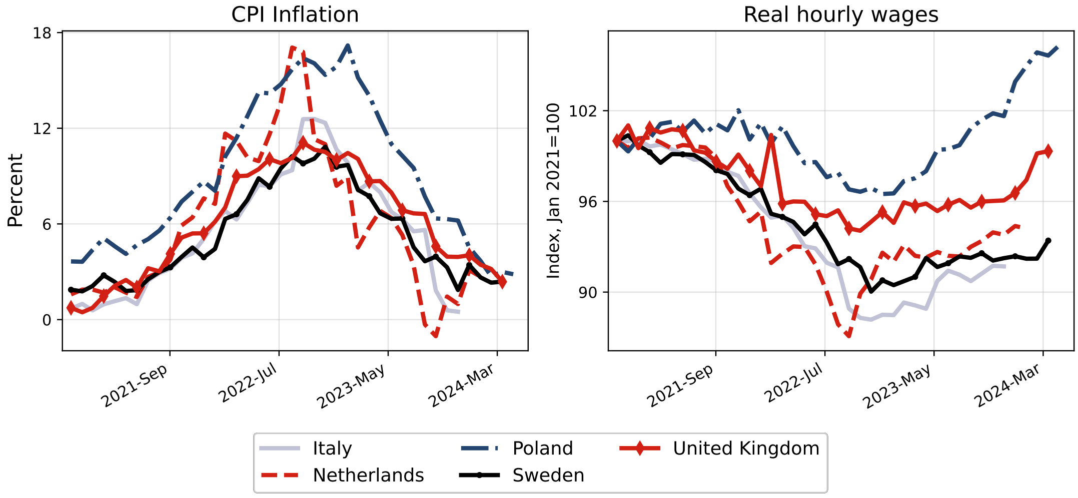


Figure 1: CPI inflation and real wages in selected European countries.

Left panel: Annual CPI inflation for selected countries obtained from the OECD. Right panel: Real hourly wages in manufacturing, defined as nominal hourly wages deflated by the consumer prices index, both obtained from the OECD. This series is indexed to 100 in January 2021.

I consider a New Keynesian model for a small open economy featuring incomplete markets (Bewley 1986; Imrohoroğlu 1989; Huggett 1993; Aiyagari 1994), and sticky nominal wages (Erceg et al. 2000). To flesh out the transmission from inflation to aggregate demand I first consider a stylized scenario where the central bank implements a ”neutral” monetary policy amounting to keep the real interest rate constant (Auclert et al. 2023b), and study a shock which is purely redistributive in nature: A domestic markup shock. In the representative agent model with a fixed real interest rate, this shock has no effect on aggregate demand and output because the entire transmission occurs through the monetary policy response to inflation, see e.g. Bodenstein et al. (2013) or Auclert et al. (2023a). I establish that a domestic markup shock which increases inflation has a contractionary effect on aggregate demand when wages are sticky and the aggregate marginal propensity to consume (MPC) out of labor income

[1]See e.g. Schnabel (2022).

is greater than that of profits. A corollary of this is the representative agent model - which typically features an equal MPC out of labor income and profits - features a zero response to markup shocks. This result arises since markup shocks are by nature purely redistributive shocks. Building on the insights from a markup shock, I consider a more general shock which in addition to the redistributive effects discussed above also has direct effects: A foreign cost-push shock in the form of an increase in the price of imported materials used in production. Unlike the markup shock, this shock is not purely redistributional within the domestic economy since it also acts as a transfer from the small open economy to the rest of the world. Hence this shock has a direct effect which can affect aggregate demand even when labor income and profits are distributed equally and their MPCs are positive. I show, however, that in the empirically relevant case in which the MPC out of labor income exceeds that of profits, the majority of the transmission to domestic demand still occurs through the distributional channel arising from a decline in the real wage.

The effect on real income also has normative implications in the HANK model. Since poorer households generally exhibit a higher marginal utility of consumption, the social planner implicitly weights the consumption of the these households higher in the optimal allocation under a utilitarian welfare function. Because these households simultaneously are less able to smooth consumption in response to income shocks due to borrowing constraints, this leads to a larger incentive for the planner to stabilize real income than in the corresponding representative agent model. I compute the optimal monetary policy response under commitment to a foreign cost-push shock in a quantitative HANK model. In the baseline small open economy model I find that the optimal monetary policy response is *more* hawkish compared to the optimal policy with a representative agent. This is unlike the results obtained in the literature on optimal policy in closed, economy HANK models, which tend to find a more accommodating policy in HANK (Bhandari et al. 2021; Smirnov 2022; Bilbiie 2024), or roughly the same response as in RANK (McKay and Wolf 2023; Davila and Schaab 2023). I show that the optimal monetary policy response depends on a non-trivial interaction between incomplete markets (i.e. income inequality and borrowing frictions), the exchange rate pass-through to import prices and the sensitivity of net exports to exchange rates. The main mechanism is as follows: In the open economy the social planner exhibits market power over the supply of domestic goods through aggregate demand management, and may therefore manipulate the terms of trade (Corsetti and Pesenti 2001). When net exports are relatively inelastic w.r.t relative price movements - as is the case empirically (Boehm et al. 2023) in the short run - it is possible for the social planner to engineer a large appreciating of the domestic currency (which reduces import prices) without reducing net exports too dramatically through expenditure switching. I demonstrate that this mechanism lead to different optimal policy allocations in HANK and RANK. In HANK the planner prefers to engineer this appreciation in order to reduce domestic inflation and stabilize real wages, which has large welfare gains for households who are likely to be borrowing constrained and therefore cannot smooth consumption in response to declines in real income. In RANK this motive is not present since agents are able to smooth consumption in the absence of borrowing constraints. In extension I show that optimal monetary policy in HANK is even more hawkish policy in the presence of foreign currency debt (De Ferra et al. 2020) and cyclical income risk (Bilbiie 2020, Acharya et al. 2023).

1.1 Related literature

The paper connects to several strands of literature. First and foremost I relate to the literature analyzing the transmission of shocks in New Keynesian business cycle models with heterogeneous agents (HANK models). A vast number of papers have highlighted the importance of a high average MPC in the transmission of various shocks, see for instance Kaplan et al. (2018), Hagedorn et al. (2019), Auclert et al. (2020), Luetticke (2021), Auclert et al. (2023b) for closed economy models and De Ferra et al. (2020), Druedahl et al. (2022), Oskolkov (2023), Auclert et al. (2024b) for open economy models.

I also relate to the classic literature studying oil and energy price shocks in the context of a small open economy with a representative agent. Mendoza (1995) and Kose (2002) use quantitative business cycle models to evaluate the effect of foreign price shocks on domestic business cycles. Baqaee and Farhi (2024) provides a general treatment of supply shocks in an open economy. Rotemberg and Woodford (1996) study the transmission of oil price shocks in a neoclassical growth model with focus on imperfect competition. Blanchard and Gali (2007) formulate a New-Keynesian model with a real wage rigidity, and use it to explain the differences in empirical responses of the US economy to oil price shocks in the 1970s and the 2000s.

Additionally, there is small number of papers that also investigates the effects of inflationary shocks in HANK models, see Cravino et al. (2020), Yang (2022), Pieroni (2022). The closest related paper is Auclert et al. (2023a), who analyses the effects of higher energy prices on aggregate demand in a SOE HANK model. In contrast to my paper, their analysis features constant markups and do not consider optimal policies. Diz et al. (2023) highlight the role of relative price/wage stickiness in a TANK model, but focus solely on demand shocks. Bobasu et al. (2024) study the transmission of energy price shocks in a HANK model with focus on non-homothetic preferences over domestic and foreign goods.

My paper is also related to a string of papers that compute optimal monetary policy in HANK models (Bhandari et al. 2021, Le Grand et al. 2021, Nuño and Thomas 2022, Acharya et al. 2023, Davila and Schaab 2023, McKay and Wolf 2023 and Acharya and Challe 2024). All of these papers focus solely on the closed economy, the one exception being Acharya and Challe (2024), who take a more analytical approach with a focus on cyclical income inequality, whereas my focus when computing optimal monetary policy is more quantitative in nature.[2] Additionally, they focus on shocks to productivity and capital flows, whereas I focus on cost-push shocks. Chan et al. (2024) study energy price shocks in a TANK model and find that the optimal monetary policy is less contractionary with borrowing constraints. The main differences between our models are that their model features no inequality, full home bias in consumption and a positive net foreign asset position, and is therefore less sensitive to the exchange rate channel studied in Auclert et al. (2024b). Chen et al. (2023) study optimal monetary policy in a two-country TANK model. Corsetti and Pesenti (2001), Benigno and Benigno (2003) and Faia and Monacelli (2008) study optimal monetary policy in open economies featuring terms-of-trade externalities with a representative agent.

2 Model

This section describes the baseline heterogeneous agent New Keynesian model used in the rest of the paper. I consider a small open economy inhabited by a continuum of households and firms. Households consume domestic and foreign tradeable goods and may save in foreign or domestic government bonds due to free capital flows similar to Obstfeld and Rogoff (2000) and Gali and Monacelli (2005), but are subject to idiosyncratic earnings risk and credit frictions as in Bewley (1986), Imrohoroğlu (1989), Huggett (1993) and Aiyagari (1994). Domestic firms produce a tradeable good using labor and imported materials subject to nominal price frictions. Unions have market power and decide on the labor supply of households subject to nominal wage frictions. There is no aggregate risk; only unanticipated aggregate shocks which materialise at date zero, after which all agents in the economy have perfect foresight with respect to aggregate variables (MIT shocks).[3]

[2] This is also reflected in the formulation of the household block. Acharya and Challe (2024) use a Blanchard-Yaari type model augmented with hand-to-mouth consumers which permit analytical aggregation. I use the standard incomplete markets model following Bewley (1986), Imrohoroğlu (1989), Huggett (1993) and Aiyagari (1994).

[3] This assumption does not matter for the results in Sections 3-4 which utilize first-order approximations around the deterministic steady state, but is necessary for the computation of optimal policy in Section 5.

2.1 Households

While the analytical results I present in section 3-4 are general and require almost no structure on the household problem, it is useful to have a baseline model in mind for the numerical examples and the quantitative analysis. The economy consists of a continuum of households with unit measure. Households are subject to idiosyncratic income risk e (described in detail in the calibration section). Households can save in a domestic mutual fund but cannot insure against idiosyncratic risk due to incomplete credit markets. A household with existing asset position a and idiosyncratic earnings e chooses consumption c and savings a' optimally by solving the recursive problem:

$$V_t(e,a) = \max_{c,a'} u\left(c\right) - \nu\left(L_t\right) + \beta \mathbb{E}\, V_{t+1}(e',a') \tag{1}$$

$$\text{s.t.}$$

$$c + a' = \left(1 + r_t^a\right) a + Z_t e + g\left(e\right) \Pi_t - \tau\left(e\right), \tag{2}$$

$$a' \geq \underline{a}, \tag{3}$$

where $Z_t \equiv \frac{W_t L_t}{P_t}$ denotes real labor income (the product of the real wage W_t/P_t and labor supply L_t), r_t^a is the real return on assets, Π_t denotes real profits received from firms, and $\tau\left(e\right)$ denote a lump sum tax raised by the government. Profits in (2) are distributed according to a function $g\left(e\right)$ which depends on earnings e and integrates to 1, $\int g\left(e\right) \mathrm{d}\mathcal{G}_t = 1$, where $\mathcal{G}_t$ denotes the time t endogenous distribution of households over states.[4]

The functional forms of the utility functions are given by:

$$u\left(c_t\right) = \ln c_t, \quad \nu\left(L_t\right) = \xi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}},$$

where ξ measures the disutility of supplying labor, and φ denotes the Frisch elasticity. Aggregates are defined by:

$$C_t = \int c_t\left(e,a\right) \mathrm{d}\mathcal{G}_t\left(e,a\right), \qquad A_t = \int a_t\left(e,a\right) \mathrm{d}\mathcal{G}_t\left(e,a\right) \tag{4}$$

Consumption basket. Consumption of goods C_t is a CES aggregate over foreign and domestic goods with elasticity of substitution η:

$$C_t = \left[\alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} + \left(1-\alpha\right)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

[4] In Section 3 I will vary the distributional rule $g\left(\bullet\right)$ to illustrate the role of profit incidence in the model.

The demand functions for $C_{H,t}, C_{F,t}$ are then given by:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \tag{5}$$

where $P_{H,t}, P_{F,t}$ are the prices of domestic and foreign tradeables in domestic currency units, and the CPI (P_t) is defined by:

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{6}$$

I assume a law of one price such that $P_{F,t} = \mathcal{E}_t P^*_{F,t}$, where $P^*_{F,t}$ denotes foreign exports prices and $\mathcal{E}$ denotes the nominal exchange rate. Note that with this convention, an increase in $\mathcal{E}$ indicates a nominal *depreciation* as in Gali and Monacelli (2005).

2.2 Supply side

The supply side is mostly standard. Firms produce output Y_t using labor and materials subject to monopolistic competition. Materials may be either purchased domestically or imported from abroad.

Representative competitive producer. There is a representative competitive producer who aggregates the output of a continuum of monopolistically competitive firms using CES technology with elasticity of substitution ϵ^p:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon^p-1}{\epsilon^p}} \mathrm{d}i\right]^{\frac{\epsilon^p}{\epsilon^p-1}}.$$

Optimization implies a standard demand curve for differentiated products:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon^p} Y_t, \tag{7}$$

where $P_{H,t}$ is the price of home output.

Monopolistically competitive firms. The representative competitive producer purchases goods from a continuum of monopolistically competitive firms. In anticipation of a symmetric equilibrium, I drop the index i from here on out. The production technology of these firms is described by a CES function, where output Y_t is produced using labor L_t and intermediate goods X_t:

$$Y_t = \left[\alpha_L^{\frac{1}{\nu}} L_t^{\frac{\nu-1}{\nu}} + (1-\alpha_L)^{\frac{1}{\nu}} X_t^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}. \tag{8}$$

Labor is rented from unions at the nominal wage W_t. Denoting by $P_{X,t}$ the price of intermediate goods, the first-order conditions for input demands are:

$$L_t = \alpha_L \left(\frac{W_t}{MC_t} \right)^{-\nu} Y_t,$$
$$X_t = (1-\alpha_L) \left(\frac{P_{X,t}}{MC_t} \right)^{-\nu} Y_t,$$

where MC_t denotes the nominal marginal cost of producers. The overall input of intermediate goods X_t is a CES of materials produced by firms themselves ($X_{H,t}$) and from foreign producers ($X_{F,t}$), i.e. imports:

$$X_t = \left[\alpha_X^{\frac{1}{\psi}} X_{H,t}^{\frac{\psi-1}{\psi}} + (1-\alpha_X)^{\frac{1}{\psi}} X_{F,t}^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}. \tag{9}$$

Domestic intermediate goods are purchased at the relevant output price while foreign intermediate inputs cost $P^*_{X,t}$ in foreign currency. The implied demand for intermediate inputs is:

$$X_{H,t} = \alpha_X \left(\frac{P_{H,t}}{P_{X,t}} \right)^{-\psi} X_t, \tag{10}$$

$$X_{F,t} = (1-\alpha_X) \left(\frac{\mathcal{E}_t P^*_{X,t}}{P_{X,t}} \right)^{-\psi} X_t. \tag{11}$$

In Sections 4-5 I will study a shock to the price of foreign imports $P^*_{X,t}$ as a source of cost-push shocks.

Pricing friction. Firms choose prices and quantities subject to the demand function (7) and subject to a price adjustment cost a lá Rotemberg (1982) given by $\frac{\theta^P}{2}\pi^2_{H,t}Y_t$. Optimization yields a New Keynesian Philips-curve relating inflation $\pi_{H,t}$ to real marginal costs $mc_t = MC_t/P_t$ and markups:

$$\pi_{H,t}(1+\pi_{H,t}) = \kappa^P \left(mc_t - \frac{P_{H,t}}{P_t}\frac{1}{\mu} \right) + \beta \pi_{H,t+1}(1+\pi_{H,t+1}), \tag{12}$$

where $\kappa^P \equiv \frac{\epsilon^P}{\theta^P}$ denotes the slope of the Philips-curve, and μ is the steady state markup.

Profits. Profits of domestic firms - measured in units of the CPI - are given by:

$$\Pi_t = \frac{P_{H,t}}{P_t} Y_t - \frac{W_t}{P_t} L_t - \frac{P_{X,t}}{P_t} X_t - \frac{\theta^P}{2} \pi_{H,t}^2 Y_t \tag{13}$$

2.3 Labor supply and wage setting

Labor supply is determined by unions as in Erceg et al. (2000), Schmitt-Grohé and Uribe (2005). There is a continuum of unions, and each household i provides $\ell_{i,t}^k$ hours of work to union k. Total labor supply of household i is then $\ell_{i,t} = \int \ell_{i,t}^k \, \mathrm{d}k$. Each union assembles individual labor supply to a union-specific task $L_t^k = \int e_{i,t} \ell_{i,t}^k \, \mathrm{d}i$, and aggregate labor supply is assembled from these union-specific tasks using a CES technology:

$$L_t = \left(\int \left(L_t^k \right)^{\frac{\epsilon^W - 1}{\epsilon^W}} \mathrm{d}k \right)^{\frac{\epsilon^W}{\epsilon^W - 1}},$$

where $\epsilon^W > 0$ is the elasticity of substitution between labor types. Union k maximizes the discounted sum of future utility of its members, less a virtual Rotemberg adjustment cost on nominal wages:

$$\sum_{t=0}^{\infty} \beta^t \left(\int \left\{ u\left(c_t\left(e,a\right)\right) - v\left(L_t\right) \right\} \mathrm{d}\mathcal{G}_t\left(e,a\right) - \frac{\theta^W}{2} \left(\frac{W_t^k}{W_{t-1}^k} - 1 \right)^2 \right).$$

The problem yields a symmetric solution such that all unions choose the same wage, and all households supply the same amount of labor within each sector. The solution is characterized by the following New Keynesian wage Phillips curve:

$$\pi_t^W \left(1 + \pi_t^W\right) = \kappa^W \left\{ \frac{v'\left(L_t\right)}{U'\left(C_t\right) w_t} \mu^W - 1 \right\} + \beta \pi_{t+1}^W \left(1 + \pi_{t+1}^W\right), \tag{14}$$

where $U'\left(C_t\right) = \int e u'\left(c_t\left(e,a\right)\right) \mathrm{d}\mathcal{G}_t\left(e,a\right)$ denotes the aggregate, productivity-weighted marginal utility of consumption, the wage markup is $\mu^W \equiv \frac{\epsilon^W}{\epsilon^W - 1}$, and the slope is defined as $\kappa^W = \frac{\epsilon^W}{\theta^W}$.

2.4 Financial assets and capital flows

The assets of domestic households are administrated by a mutual fund which invests in either government domestic bonds B in fixed supply or foreign bonds B^* in infinite supply. The foreign bond pays i_t^* in foreign currency units whereas domestic bonds pay i_t which is the nominal rate set by the domestic central bank. Free capital flows then implies a nominal UIP condition $1 + i_t = \left(1 + i_t^*\right) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$. Defining the real exchange rate $Q_t = \frac{\mathcal{E}_t}{P_t}$ we may rewrite this as the real UIP condition:

$$1 + r_t = (1 + i^*) \frac{Q_{t+1}}{Q_t}, \tag{15}$$

where $r_t = \frac{1+i_t}{1+\pi_{t+1}} - 1$ is the ex-ante real interest rate. Note that equilibrium in the asset market implies $A_t = B_t^* + B$. I define the net foreign asset position as the difference between domestic asset A_t and the supply of domestic bonds B, $NFA_t = A_t - B$, which also implies $NFA_t = B_t^*$.

2.5 Monetary policy

The domestic central bank controls the nominal interest rate i_t, which is related to the ex-ante real interest rate through the Fisher relation $1 + r_t = \frac{1+i_t}{1+\pi_{t+1}}$. In the baseline analysis I assume a neutral monetary policy stance which aims to keep the domestic ex-ante real rate constant, $r_t = r$ (see Auclert et al. (2023b) for a similar approach). This may be interpreted as a Taylor rule with coefficient 1 on expected inflation. I return to the role of monetary policy in Section 5.

2.6 Government

The government supplies bonds B and raises taxes τ_t to pay interest on issued bonds. The budget constraint is given by:

$$\tau_t = r_t B.$$

2.7 Exports

Foreign demand for domestic goods $C_{H,t}^*$ is a standard Armington demand function:

$$C_{H,t}^* = \alpha^* \left(\frac{P_{H,t}^*}{P_{F,t}^*} \right)^{-\eta}. \tag{16}$$

I assume a law of one price such that $P_{H,t}^* = \frac{P_{H,t}}{\mathcal{E}_t}$.

2.8 Market clearing and equilibrium

Market clearing in the economy is given by:

$$Y_t - X_{H,t} = C_{H,t} + C_{H,t}^* + \frac{P_t}{P_{H,t}} \frac{\theta^P}{2} \pi_{H,t}^2 Y_t. \tag{17}$$

The general equilibrium of the model is defined as follows:

Definition 1 (Equilibrium in the small open economy.). *Given a sequence of shocks $\{\mu_t, P^*_{X,t}\}$, an initial household distribution over assets and earnings $\mathcal{G}_0(a,e)$, and an initial portfolio allocation between foreign and domestic bonds, a competitive equilibrium in the domestic economy is a path of household policies $\{c(a,e), a'(a,e)\}$, distributions $\mathcal{G}_t(a,e)$, prices:*

$$\left\{E_t, Q_t, P_t, P_{H,t}, P_{F,t}, W_t, P_t^X, i_t, r_t\right\}$$

and quantities:

$$\{C_t, C_{H,t}, C_{F,t}, A_t, Y_t, X_t, X_{H,t}, X_{F,t}, L_t, \Pi_t, NFA_t, B_t^*\}$$

such that all households and firms optimize, the central bank sets monetary policy according to the chosen rule, and the goods market clearing condition eq. (17) *holds, while the asset market clearing condition $A_t = B_t^* + B$ holds residually by Walras's law.*

2.9 Representative agent economy

I compare my results for the HANK model to those obtained with a textbook representative agent model. Here aggregate consumption follows the Euler equation:

$$u'(C_t) = \beta\left(1 + r_{t+1}^a\right) u'(C_{t+1})$$

with $\beta = \frac{1}{1+r}$ in steady state. Additionally, the marginal utility term in the wage Philips-curve is replaced by the marginal utility of aggregate consumption ($U'(C_t) = 1/C_t$ with log preferences).

2.10 Calibration

I calibrate the model to target the average small open economy in the OECD.[5] Table 1 displays the calibration. For the household block the Frisch elasticity of labor supply is set to a standard values of 0.5. The discount factor β is calibrated to match an aggregate MPC out of a uniform lump sum transfer of 0.51 following Fagereng et al. (2021). In Figure 2a I plot the dynamic MPCs in the model following a one time unexpected transfer against the estimated MPCs from Fagereng et al. (2021). The model overall replicates the empirical evidence well. In Figure 2b I plot the corresponding dynamic MPCs following an increase in aggregate *real labor income* Z. The first-year MPC is slightly lower than the transfer-MPC at 0.38 because Z loads more on rich households

[5]The sample used is described in detail in Druedahl et al. (2022).

who have lower MPCs. The MPC out of labor income will be a central object in the analysis in the remainder of the paper.

For the earnings process e I follow Bayer et al. (2019) and Gornemann et al. (2021) in introducing an entrepreneurial (or "superstar") state in the earnings process e. In particular, define e as the normalized version of $\tilde{e}_{i,t}$, $e_{i,t} = \frac{\tilde{e}_{i,t}}{\int \tilde{e}_{i,t}\,\mathrm{d}i}$, where:

$$\tilde{e}_t = \begin{cases} \exp\left\{\rho^e \tilde{e}_{t-1} + \varepsilon^e_t\right\} & \text{with probability } 1-\upsilon \text{ if } e_{t-1} \neq 0 \\ 1 & \text{with probability } \iota \text{ if } e_{t-1} = 0 \\ 0 & \text{Else,} \end{cases}$$

where ε^e_t is mean zero normal innovation. This captures that idiosyncratic income of workers follow an AR(1) process in logs with persistence ρ^e and standard deviation σ^e. With probability υ they become entrepreneurs and receive no labor income, but instead earn firm profits. With probability ι they exit the entrepreneurial state and return to the worker state, starting with median earnings. I fix the probability of leaving the entrepreneurial state ι to $1/16$ per quarter as in Bayer et al. (2019), so as to match the probability of dropping out of the top income percentile is the US (Guvenen et al. 2014). The assumption that entrepreneurs receive the entirety of firm profits implies the following form of $g(e)$[6]:

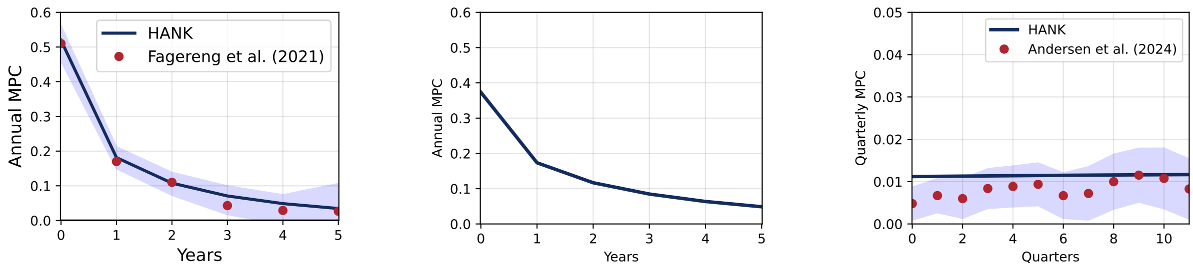
$$g(e) = \begin{cases} 0 & \text{if } e \neq 0 \\ \frac{1}{\pi^e(e)} & \text{if } e = 0 \end{cases},$$

where π^e is the CDF of e. I calibrate the entry probability υ to match an aggregate, annual MPC out of profits of 4%. Chodorow-Reich et al. (2021) estimate an annual MPC out of stock returns of 3.2% using US data. Andersen et al. (2024) estimate an annual MPC of 4% using Danish data. Figure 2c plots the aggregate quarterly MPC out of a one-time increase in profits against the estimated, dynamic response from Andersen et al. (2024). The overall response is relatively flat because rich households - who are the ones that have claims on firm profits - act as permanent income households, and thus smooth consumption extensively. Overall, this fits the empirical evidence well.

For the tax function $\tau_t(e)$ I posit that households are taxed in proportion to total income (both labor and profits):

$$\tau_t(e) = \tau_t \frac{\mathbb{1}_{\{e=0\}} \frac{\Pi_t}{\pi^e(e)} + e}{\int \left(\mathbb{1}_{\{e=0\}} \frac{\Pi_t}{\pi^e(e)} + e\right) \pi^e(e)\,\mathrm{d}e}.$$

[6]The results are robust to alternative modelling choices, such as having profits paid out in proportion to wealth as in the mutual fund setup of Hagedorn et al. (2019).



(a) MPC - uniform transfer (b) MPC - labor income (c) MPC - profits

Figure 2: Marginal propensities in the calibrated model

On the supply side I fix the steady state markup at a standard value of 20%, as is common in the literature. I follow Nakamura and Steinsson (2008) in setting the slope of the price Philips-curve to $\kappa^P = 0.1$. For the elasticities of substitution in production I follow the estimates in Boehm et al. (2019), who find evidence of little substitution between factors in the short run following a supply shock. This leads me to set the elasticity of substitution between labor and materials to their preferred estimate $\nu = 0.03$. This rather low value is consistent with related papers that study inflationary shock such as Auclert et al. (2023a), Chan et al. (2024). I discuss the implications of a higher elasticity in Section 4. Regarding the elasticity of substitution between foreign and domestic materials I fix this at $\psi = 0.5$ also following Boehm et al. (2019), which is a standard value in the literature. Regarding the cost-structure I calibrate the input share of labor α_L to match the average cost-share of labor in the sample OECD countries. This yields residually spending on total materials X_t.

For the unions I fix the slope of the wage Philips-curve to $\kappa^W = 0.01$ following Sbordone (2006) and Schmitt-Grohé and Uribe (2006), reflecting relatively sluggish nominal wage adjustment. The wage markup is set equal to the markup of firms (20%) as in Smets and Wouters (2007).

The remaining parameters of the model concern trade. I set the elasticity of import/export demand η to 2, which is standard in the literature.[7]. I calibrate the size of foreign economy α^* such that steady state exports amounts to 42% of GDP. I assume the net-foreign asset position (and net exports) are 0 in steady state. I fix the share of imports in domestic household's consumption basket at 25%, $\alpha = 0.25$ following Christiano et al. (2011). The share of imported materials by firms is then calculated residually such that aggregate imports constitute 42% of GDP. This yields $\alpha_X = 0.83$, such that firms imports 17% of materials.

[7] I return to the exact value of this elasticity in Section 5

Parameter	Description	Value	Source/Target
Households			
φ	Frisch	0.5	Chetty et al. (2011)
β	Discount factor	0.981	MPC = 0.51 (Fagereng et al. (2021))
ρ^e	Persistence of idiosyncratic shocks	0.966	Floden and Lindé (2001)
σ^e	Std. dev of idiosyncratic shocks	0.13	Floden and Lindé (2001)
$\underline{a}$	Borrowing limit	0	Standard value
Firms			
μ	Firm markup	1.2	Standard value
μ^W	Wage markup	1.2	Standard value
κ^P	Slope of Philips curve	0.1	Nakamura and Steinsson (2008)
κ^W	Slope of wage Philips curve	0.01	Sbordone (2006) and Schmitt-Grohé and Uribe (2006)
ν	EOS between labor and intermediate goods	0.03	Boehm et al. (2019)
ψ	EOS foreign and domestic intermediate goods	0.5	Boehm et al. (2019)
α_L	Spending on labor input	0.35	$\frac{WN}{P^X X+WN}=0.35$ (OECD average)
α_X	Spending on domestic materials	0.83	Imports = 42% of GDP (OECD average)
Trade			
α	Imports of final goods	0.25	Christiano et al. (2011)
α^*	Exports	0.42	$NX = 0$ in steady state
η	EOS between foreign and domestic goods	2.0	Standard

Table 1: Calibration

3 Transmission of Inflationary Shocks

To highlight the main channel investigated in the paper - the relative effects of wage/price stickiness and the interaction with limited insurance and high MPCs - the initial analysis focuses on a pure inflationary shock, namely a temporary increase in the desired markups of domestic firms. In Section 4 I extend the analysis to a more general cost-push shock.

For tractability in the derivation of analytical results, I focus on a special case where labor and materials are perfect complements, $\nu \to 0$. Given the estimates from Boehm et al. (2019), which underlie the calibration used, this is not an extreme approximation when focusing on short-run dynamics. Secondly, I assume that only imported materials are used in production along with labor, $\alpha_X = 0$.[8] Finally, I assume that there are no wealth effects on labor supply. This is in line with empirical evidence which usually find small wealth effects; see Galí et al. (2012) for a way to micro-found this. These simplifying assumptions are only used in the analytical derivations, and do not apply in any of the numerical illustrations, figures which use the full quantitative model from Section 2.[9]

[8] This assumption is not important for the analytical results, but does help with exposition.

[9] All analytical results apply approximately in the full model, reflecting that these assumptions constitute only a minor deivaiton from the quantitative model.

3.1 Markup shocks

I start by considering a markup shock - an exogenous change in μ in the Philips-curve (12) - which is often found to explain the majority of variation in inflation at the business cycle frequency in closed economies.[10] For the baseline analysis I assume a neutral stance of the domestic central bank in the sense that they keep the domestic real rate constant (as in Auclert et al. 2023b) at the steady state level, $r_t = r$. Note that in the standard 3-equation New-Keynesian model the entire transmission of markup shocks to the real economy derives from the response of the central bank to inflation.[11] This is because domestic demand is driven entire by intertemporal substitution, or what Kaplan et al. (2018) call the "direct effect" of monetary policy. Hence the assumption of a constant real rate eliminates the main transmission channel present in the standard NK model to more clearly highlight the distributional dynamics which are the focus here.

The analysis is centered around the goods market clearing condition (17). Linearizing and applying the assumption of a constant real interest rate, which implies a constant real exchange rate $Q_t = Q$ by the real UIP condition (15), yields that changes in domestic output equals the change in domestic consumption spend on home goods:[12]

$$dY_t = (1-\alpha)\, dC_t, \tag{18}$$

where $dx_t = x_t - x$ represent deviations from steady state for some variable x. Given a constant real rate the aggregate consumption function $\mathcal{C}_t$ depends only on the sequences of real labor income $\{Z_s\}_{s=0}^{\infty} = \left\{\frac{W_s}{P_s} L_s\right\}_{s=0}^{\infty}$ and profits $\{\Pi_s\}_{s=0}^{\infty}$, i.e. $C_t = \mathcal{C}_t\left(\{Z_s, \Pi_s\}_{s=0}^{\infty}\right)$. Stacking variables in vectors $d\mathbf{C} = (dC_0, dC_1, \ldots)$ the linearized consumption function can be written as $d\mathbf{C} = \mathbf{M}^Z d\mathbf{Z} + \mathbf{M}^{\Pi} d\mathbf{\Pi}$ where $\mathbf{M}^Z, \mathbf{M}^{\Pi}$ are the sequence-space Jacobians of aggregate consumption w.r.t labor income and profits respectively:[13]

$$\mathbf{M}^Z = \begin{bmatrix} \frac{\partial C_0}{\partial Z_0} & \frac{\partial C_0}{\partial Z_1} & \cdots \\ \frac{\partial C_1}{\partial Z_0} & \frac{\partial C_1}{\partial Z_1} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \quad \mathbf{M}^{\Pi} = \begin{bmatrix} \frac{\partial C_0}{\partial \Pi_0} & \frac{\partial C_0}{\partial \Pi_1} & \cdots \\ \frac{\partial C_1}{\partial \Pi_0} & \frac{\partial C_1}{\partial \Pi_1} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}.$$

Here the entry $M_{0,0}^Z$ corresponds to the quarterly MPC out of labor income and so forth. Note that the conventional MPC estimated in the literature is the change in consump-

[10] See Smets and Wouters (2003), Smets and Wouters (2007), Del Negro et al. (2015).

[11] This is true to first-order. At second-order or higher inflation generates a resource loss from adjustment (Rotemberg) or misallocation (Calvo) depending on the specification of the pricing friction.

[12] See appendix A.1 for the derivation.

[13] See Auclert et al. (2023b) for more details on the existence of the consumption function, and the sequence-space Jacobians.

tion given a one-time unexpected lump-sum transfer (e.g. Shapiro and Slemrod 2003; Johnson et al. 2006; Fuster et al. 2021; Fagereng et al. 2021). As we shall see the relative level of $\mathbf{M}^Z, \mathbf{M}^\Pi$ will have important implications for the transmission of markup shocks. Combining the linearized consumption function with the definition of profits and goods market clearing I obtain the following proposition:

Proposition 1 (Equilibrium relationship between output and profits)**.** *Given a sequence of markup shocks $\{\mu_s\}_{s=0}^{\infty}$ the equilibrium relation between output and profits is given by:*

$$\frac{1}{1-\alpha} d\mathbf{Y} = \underbrace{\alpha_L \mathbf{M}^Z d\mathbf{Y}}_{\text{Multiplier}} - \underbrace{\left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] d\mathbf{\Pi}}_{\text{Distributional channel}} \tag{19}$$

Proof: appendix A.2.

Proposition 1 implies that the response of output to a markup shock depends on a multiplier term and a distributional term relating to changes in profits. Given a constant real rate the distribution effects are the fundamental source of propagation following the shock. In particular, with a representative agent - which typically features $\mathbf{M}^Z = \mathbf{M}^\Pi$ - the distributional effect is zero, and the solution to (19) is $d\mathbf{Y} = \mathbf{0}$. Hence the proposition highlights exactly why a markup shock has no effects (besides the effect through monetary policy) in the basic NK model featuring a representative agent. The same insight holds in more general HANK models which features an equal incidence of labor income and profits across the population. In this case the marginal propensities may be positive $\mathbf{M}^Z > 0, \mathbf{M}^\Pi > 0$ but to the extent that they are equal, $\mathbf{M}^Z = \mathbf{M}^\Pi$, redistribution is neutral in the aggregate, and the markup shock has no effect on aggregate demand. This is summarized in corollary 1:

Corollary 1. *If the model features an equal incidence of labor income and profits, and therefore delivers equal marginal propensities to consume out of labor income and profits, $\mathbf{M}^Z = \mathbf{M}^\Pi$, the markup shocks have no effect on real output, $d\mathbf{Y} = \mathbf{0}$, under a neutral monetary policy stance, $d\mathbf{r} = \mathbf{0}$.*

Moving onto the more general case where $\mathbf{M}^Z \neq \mathbf{M}^\Pi$, we see that if the MPC out of labor income is greater than that of profits, as the data suggests, $\mathbf{M}^Z > \mathbf{M}^\Pi$, an increase in firm markups, which increase profits $d\mathbf{\Pi} > 0$, will suppress aggregate demand and generate a contraction in terms of domestic output, $d\mathbf{Y} < 0$.

Figure 3 displays the responses to a markup shock that increases inflation by 1% on impact in a baseline RANK ($\mathbf{M}^Z = \mathbf{M}^\Pi = \mathbf{0}$) model, a HANK model with equal incidence of labor income and profits ($\mathbf{M}^Z = \mathbf{M}^\Pi > \mathbf{0}$), and a HANK model with unequal incidence (the baseline model - $\mathbf{M}^Z > \mathbf{M}^\Pi > \mathbf{0}$). Specifically, the Figure plots the responses of real labor income Z, aggregate output Y, inflation π and real profits Π.

For both the RA and HA model the increase in desired markups causes firms to raise prices further above marginal costs hence generating inflation. Given nominal wage frictions, this drives down the real wage and increases firm profits. The two models then differ in how aggregate demand and output respond to these changes. In the RA model (left panel) with a neutral monetary policy stance aggregate consumption and output does not respond since the MPC out of transitory income changes is zero. The middle panel shows that even with positive and large MPCs the response of aggregate demand and output is zero in HANK when the incidence of labor income and profits are equal (corollary 1). Hence the RA and HA models are equivalent for a domestic markup shocks in this special case. If we consider the empirically realistic case where the aggregate MPC out of labor income is greater than that of profits (rightmost panel) the redistribution caused by inflation is not demand-neutral and the model therefore delivers a significant drop in domestic demand and output, i.e. stagflation.

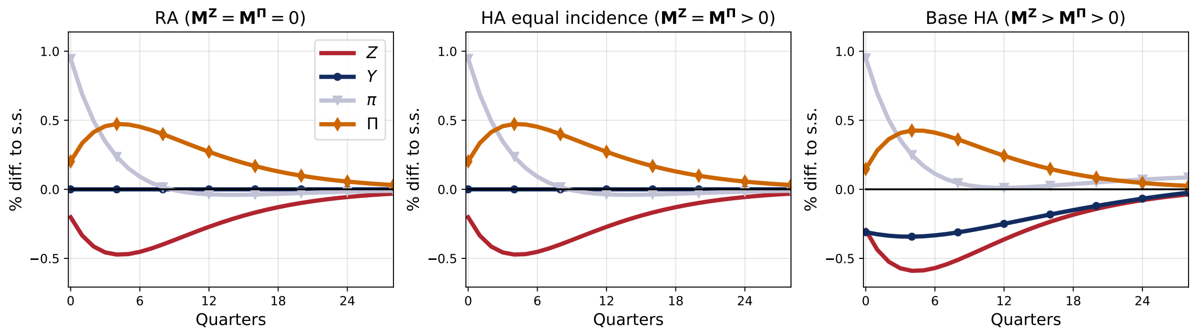


Figure 3: Effect of markup shock in RANK and HANK

Note: Impulse responses to an AR(1) shock to the domestic markup with persistence 0.8. The shock is normalized such that inflation increase 1% on impact. The Figure shows real labor income Z (red), real output Y (dark blue), CPI inflation π (grey) and real profits Π (orange).

Two-agent model. The transmission occurring through factor income redistribution is very clear in the case of a two-agent model à la Galí et al. (2004) where the matrices $\mathbf{M}^Z, \mathbf{M}^\Pi$ have closed form solutions. Assume that a share λ of households are financially constrained with the remaining $1-\lambda$ share being permanent-income type households (implying an aggregate MPC of λ). Constrained households receive a share δ of aggregate profits such that $\delta = \lambda$ implies an equal incidence of profits across the population. These assumptions imply $\mathbf{M}^Z = \lambda \mathbf{I}$ and $\mathbf{M}^\Pi = \frac{\delta}{\lambda}\lambda \mathbf{I} = \delta \mathbf{I}$. Equation (19) then takes the simple form:

$$\frac{1}{1-\alpha} d\mathbf{Y} = \alpha_L \lambda d\mathbf{Y} - [\lambda - \delta]\, d\mathbf{\Pi},$$

from which it is evident that $\lambda = \delta$ yields $d\mathbf{Y} = 0$ and $\lambda > \delta$ implies $d\mathbf{Y} < 0$ for $d\mathbf{\Pi} > 0$.

MPCs across the population. Additional intuition for the mechanisms present in eq. (19) can be provided by re-writing it in terms of covariances as follows:[14]

$$\frac{1}{1-\alpha}d\mathbf{Y} = \alpha_L \mathbf{M}^Z d\mathbf{Y} - \left[\mathbb{C}\mathrm{ov}_i\left(\mathbf{M}_i, e_i\right) - \mathbb{C}\mathrm{ov}_i\left(\mathbf{M}_i, g\left(e_i\right)\right)\right] d\mathbf{\Pi}, \tag{20}$$

where $\mathbf{M}_i$ is the the Jacobian of consumption w.r.t a uniform lump-sum transfer for individual i in the population.

Each of the the terms here corresponds exactly to the terms in (19). Focusing on the second term on the right-hand side, eq. (20) shows that the differential $\mathbf{M}^Z - \mathbf{M}^\Pi$ corresponds exactly to the differential of two covariances: The population covariance between intertemporal MPCs and idiosyncratic income and the covariance between intertemporal MPCs and the profit incidence function.[15]. If profits tend to be distributed towards low MPC households more so than labor earnings, then eq. (20) implies that an increase in profits has contractionary short-run effects. Figure 4 displays the effects on inflation, real wages and consumption varying the covariances gap in (20), or equivalently, the difference between the Jacobians $\mathbf{M}^Z - \mathbf{M}^\Pi$ by changing the profit incidence function $g(e)$. As the profits become more equally distributed in the population the gap $\mathbf{M}^Z - \mathbf{M}^\Pi$ lessens, and the contractionary effect on consumption becomes smaller, converging to zero when $\mathbf{M}^Z = \mathbf{M}^\Pi$ as per corollary 1.

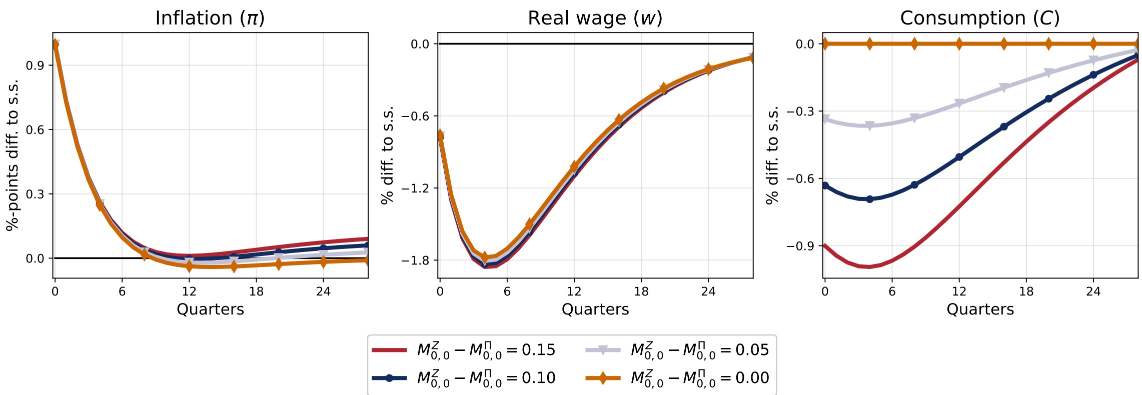


Figure 4: Responses to a markup shock with varying profit distribution across the population

3.2 General equilibrium

So far I have taken the response of aggregate profits $d\mathbf{\Pi}$ as given. Solving the model for $d\mathbf{\Pi}$ and substituting into eq. (19) characterizes the full general equilibrium solution

[14]See appendix A.3 for the derivation.

[15]For the purpose of this exposition, I assume that the covariance operator $\mathbb{C}\mathrm{ov}(\mathbf{X}, y)$ which takes a matrix $\mathbf{X}$ and a scalar y as input gives as output a matrix which is simply the element-by-element covariances between $\mathbf{X}$ and y.

for output. This operation requires writing the New-Keynesian Philips curve (12) and wage Philips curve (14) in sequence-space:

$$d\mathbf{P_H} = \boldsymbol{\kappa}^P \left(d\mathbf{mc} + d\boldsymbol{\mu}\right) \tag{21}$$

$$d\mathbf{W} = \boldsymbol{\kappa}^W \left(\frac{w}{\phi} d\mathbf{N} - d\mathbf{w}\right), \tag{22}$$

where the bold letters $\boldsymbol{\kappa}^P, \boldsymbol{\kappa}^W$ are the Philips curve pass-through matrices (Auclert et al. 2024a).[16] Note that even though these are matrices, they are proportional to the slopes of the respective Philips curves and so $\kappa^P = 0 \Rightarrow \boldsymbol{\kappa}^P = \mathbf{0}$, $\frac{\partial \boldsymbol{\kappa}^P}{\partial \kappa^P} \geq \mathbf{0}$ etc. To compactly solve for the general equilibrium response of the model it is useful to define the *pass-through matrix of markup shocks to markups* $\boldsymbol{\Theta}^\mu$ as well as the *pass-through matrix of employment to markups* $\boldsymbol{\Theta}^L$.

Definition 2. *The pass-through matrix of markup shocks to markups is defined by:*

$$\Theta^\mu \equiv \left[\mathbf{I} + \boldsymbol{\kappa}^W + \alpha_L W \boldsymbol{\kappa}^P\right]^{-1} \alpha_L W \boldsymbol{\kappa}^P. \tag{23}$$

Similarly, the pass-through matrix of employment to markups is defined by:

$$\Theta^L \equiv - \left[\mathbf{I} + \boldsymbol{\kappa}^W + \alpha_L W \boldsymbol{\kappa}^P\right]^{-1} \frac{\alpha_L W}{\phi} \boldsymbol{\kappa}^W, \tag{24}$$

where $\boldsymbol{\kappa}^P, \boldsymbol{\kappa}^W$ are the slops of the Philips curves (21)-(22), α_L is the share of labor used in production, ϕ is the Frisch elasticity, and W is the steady state wage rate.

The markup pass-through matrix $\boldsymbol{\Theta}^\mu$ captures the effect of an increase in desired markups μ (or equivalently, in marginal costs) on the markup (defined as prices over nominal marginal costs, $P_{H,t}/MC_t$) through the two Philips curves (21)-(22). If we consider simple, static Philips curves of the form $dW_t = \kappa^W \left(\frac{W}{\phi} dL_t - dw_t\right)$, $dP_t = \kappa^P dmc_t$ with only labor used as input $dmc_t = dw_t$, then the entries in $\boldsymbol{\Theta}^\mu$ are constant and given by $\frac{\kappa^P}{1+\kappa^W+\kappa^P}$. The numerator captures the direct effect on markups from an increase in μ (markups increase by the shock times the pass-through to prices κ^P), while the numerator captures the feedback loop occurring through the wage Philips curve. When prices go up due to the shock, this reduces the real wages appearing in the wage Philips curve, thereby raising nominal wages by κ^W. This raises the marginal costs of firms, which causes firms to once again raise prices by κ^P. The fixed point of this

[16] If the discount factors in the Philips-curves equal 0 then these matrices are simply given by $\boldsymbol{\kappa}^P = \kappa^P \times \boldsymbol{U}, \boldsymbol{\kappa}^W = \kappa^W \times \boldsymbol{U}$ where $\boldsymbol{U}$ is an upper-triangular matrix with ones above the diagonal, and zeros below. The more general expressions are derived in appendix A.4. Note that to simplify notation the markup shocks $d\boldsymbol{\mu} = (d\mu_0, d\mu_1, \ldots)$ in (21) are defined as $d\mu_t = \frac{\mu_t - \mu}{\mu^2}$. This is just a matter of scaling given linearity.

interaction is exactly $\boldsymbol{\Theta}^{\mu}$. With fully flexible prices the pass-through is one, while with fully flexible wages the pass-through is zero. $\boldsymbol{\Theta}^{L}$ captures the same logic following an increase in labor supply.[17] An increase in labor supply puts upwards pressure on nominal wage growth by exactly κ^W/ϕ. This raises marginal cost of firms, who in turn raise their prices by κ^P. This causes a decline in the real wage, whereby unions raise nominal wages again. The fixed point of this interaction is exactly $-\frac{\kappa^W/\phi}{1+\kappa^W+\kappa^P}$. Note that this is negative since with sticky prices an increase in inputs such as labor generates temporarily lower markups.

3.2.1 General equilibrium solution

With the notation from Definition 2 in place, the following proposition characterizes the full general equilibrium solution for output:

Proposition 2. *The equilibrium response of output $d\mathbf{Y}$ to a markup shock is:*

$$d\mathbf{Y} = -\mathcal{M}\left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] \times \boldsymbol{\Theta}^{\mu} \times d\boldsymbol{\mu}, \tag{25}$$

where the Keynesian general equilibrium multiplier $\mathcal{M}$ is:

$$\mathcal{M} \equiv \left[\left((1-\alpha)\,\alpha_L\right)^{-1}\mathbf{I} - \mathbf{M}^Z - \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right]\left(\boldsymbol{\Theta}^L - \frac{1}{\alpha_L}\frac{\mu-1}{\mu}\right)\right]^{-1}.$$

Proof: appendix A.5.

Proposition (2) highlights simultaneously the importance of the MPC differential $\mathbf{M}^Z - \mathbf{M}^\Pi$ discussed earlier as well as the importance of nominal price and wage frictions (captured in $\boldsymbol{\Theta}^{\mu}$) as they determine the response of profits $d\boldsymbol{\Pi}$.

More flexible prices ($\kappa^P \uparrow$) will lead to a larger pass-through from markups to prices. In the presence of nominal wage frictions ($\kappa^W < \infty$) this will lead to a decline in the real wage and higher profits (larger $\boldsymbol{\Theta}^{\mu}$). The drop in real wages suppress aggregate demand and output if $\mathbf{M}^Z > \mathbf{M}^\Pi$. Similarly, more flexible wages ($\kappa^W \uparrow$) will imply a small drop in real wages as any drop in real wages will be met by larger nominal wage increases according to the wage Philips curve (14). This will stabilize aggregate demand since $\mathbf{M}^Z > \mathbf{M}^\Pi$ and output fluctuations will be dampened. These mechanisms are summarized in the following proposition:

Proposition 3 (Price and Wage flexibility)**.** *Consider the response of aggregate output (25) to a markup shock. If $\mathbf{M}^Z \geq \mathbf{M}^\Pi$ then:*

[17] In this stylized example the entries of $\boldsymbol{\Theta}^{L}$ are given by $-\frac{\kappa^W/\phi}{1+\kappa^W+\kappa^P}$.

(i) Increasingly flexible prices amplify the output effects of the markup shock $\frac{\partial d\mathbf{Y}}{\partial \kappa^P} \leq 0$*, while rigid prices dampen the shock. In the limit with completely rigid prices we have* $\boldsymbol{\Theta}^\mu = \mathbf{0}$*, and the general equilibrium response of output is zero:*

$$\lim_{\kappa^P \to 0} d\mathbf{Y} = \mathbf{0} \times d\boldsymbol{\mu},$$

while with fully flexible prices there is full pass-through, $\boldsymbol{\Theta}^\mu = \boldsymbol{I}$*:*

$$\lim_{\kappa^P \to \infty} d\mathbf{Y} = -\mathcal{M}\left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] \times d\boldsymbol{\mu}.$$

(ii) If prices are not completely flexible, $\kappa^P < \infty$*, increasingly flexible wages attenuate the output effects of the markup shock* $\frac{\partial d\mathbf{Y}}{\partial \kappa^W} \geq 0$*, while more rigid wages amplify the effects of the markup shock. In the limit with completely flexible wages we have* $\boldsymbol{\Theta}^\mu = \mathbf{0}$*, and the general equilibrium response of output is zero:*

$$\lim_{\kappa^W \to \infty} d\mathbf{Y} = \mathbf{0} \times d\boldsymbol{\mu}.$$

Proof: Appendix A.6.

Figures 5 and 6 illustrate Proposition 3 numerically. Figure 5 shows that a lower degree of price stickiness amplifies the markup shock by generating a larger inflation response, which for given nominal wages generates a larger real wage drop. Hence price flexibility amplifies the redistribution from workers to firm owners which suppresses aggregate demand. With sufficiently sticky prices, inflation does not move and the shock has no effect.

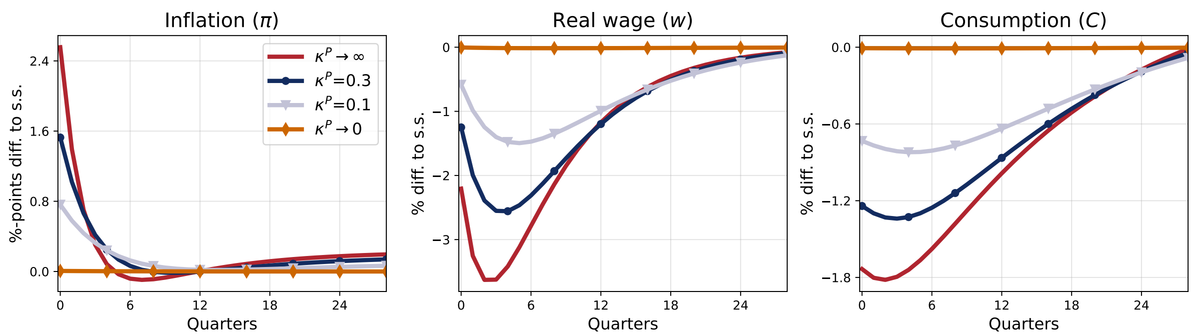


Figure 5: Transmission of markup shock with varying degree of price stickiness

Figure 6 correspondingly shows that a larger degree of wage flexibility attenuates the response to the shock. In the limit with completely flexible wages real wages remain unchanged even though the equilibrium outcome features a larger inflation response. Since real wages are unchanged the effect on aggregate demand is completely neutralized. Note that for wage flexibility to matter it is necessary that prices are not completely flexible $\kappa^P < \infty$, since in that case the markup is exogenous and simply equal to the shock $d\boldsymbol{\mu}$. In this case wage flexibility cannot affect the cyclicality of markups, and even completely flexible wages will not be sufficient to stabilize the real wage.

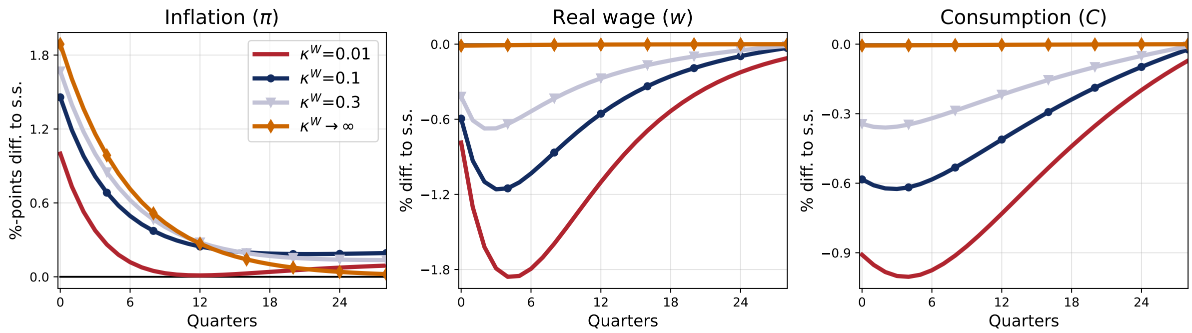


Figure 6: Transmission of markup shock with varying degree of wage stickiness

3.3 Model extensions

I here briefly cover some natural extensions to my framework, and discuss their implications for my results.

Wage indexation. Proposition 3 highlights the importance of real wage dynamics for the transmission of the shock in models with distribution effects. A natural remedy to stabilize real wages in volatile inflation environments is to index nominal wages to CPI inflation (e.g. Fischer 1976). In Appendix B.1 I solve the model with wage indexation and demonstrate that a higher degree of wage indexation dampen the output effects of the markup shock. Full indexation perfectly neutralizes the effects of the shock similarly to the case with fully flexible wages covered in Proposition 3.

Wage-price spirals. A common argument against wage indexation is that it fuels additional inflation, which may induce a wage-price spiral (Lorenzoni and Werning 2023). This mechanism is also present in my model when I introduce wage indexation, see Figure A.1. Still the full-information rational expectations (FIRE) assumption underlying the model, rules out any explosive scenarios where current inflation is driven by increasing expectations of future inflation. I here consider an extension of the model where I allow for diagnostic expectations following Bianchi et al. (2023). I find that with diagnostic expectations the effects of the output effects of the markup shock are amplified. Wage indexation can still stabilize real wages and demand, but at the cost of excessive inflation.

Fisher effects. A mechanism often brought up in the context of large inflation surges is the redistribution that occurs between borrowers and savers when debt contracts are nominal (see e.g. Auclert 2019; Nuño and Thomas 2022; Brunnermeier et al. 2023). The baseline model assumes that households save in real assets, and thereby sidesteps this channel. In Appendix B.4 I introduce nominal bonds into the economy to incorporate Fisher effects. I find that the Fisher effect dampens the initial output response due to the redistribution from savers to borrowers, but that the output effects in the following periods are amplified. The initial amplification occurs because the time 0 MPCs of borrowers are larger than the those of savers. Still, since savers tend to have larger *intertemporal* MPCs the output response in the following periods gets amplified.

4 Foreign Cost-Push Shocks

While the markup shock considered in section 3.1 is a very direct inflationary shock it is also special in the sense that it is a purely redistributive shock, and hence only affects aggregate demand when households differ (corollary 1). In this section I consider instead a foreign cost-push shock corresponding to an increase in the price of imported materials used in production ($P^*_{X,t}$ in equation (11)). Since a share of domestic resources goes abroad to pay for the imports there is an aggregate resource loss in the small open economy, and so the shock can have real effects even in the absence of heterogeneity, $\mathbf{M}^{\Pi} = \mathbf{M}^{Z}$. The aim of this section is to understand how important the distributional effect of inflation turns out to be in a setting where the shock also has more direct effects.

Preliminaries. The baseline analysis assumes that the domestic central bank keeps the domestic real rate constant, implying that the real exchange rate is also constant given the real UIP condition (15). It turns out that if we consider a foreign inflation shock that increases foreign CPI and the price of foreign inputs equally, $dP^*_{F,t} = dP^*_{X,t}$ then the assumption of a constant real rate implies that the nominal exchange rate takes the entire adjustment of the shock and the domestic economy is entirely unaffected.[18]

This result can easily be broken by considering an asymmetric shock which features a larger increase in the price of foreign inputs compared to the foreign CPI, $dP^*_{X,t} > dP^*_{F,t}$. In this section I consider the special case where $dP^*_{F,t} = 0, dP^*_{X,t} > 0$ but given linearization the shock can be interpreted as an increase import prices over and above the increase in foreign CPI.

[18]See Appendix B.5.

4.1 Analysis

Consider now a shock to the import price of materials $P^*_{X,t} > 0$. The difference between this shock and the markup shock can be seen in the following proposition, which generalizes eq. (19) to the case of a foreign cost-push shock:

Proposition 4 (Equilibrium relationship between output and profits). *Given a sequence of foreign cost-push shocks $\left\{dP^*_{X,t}\right\}_{s=0}^{\infty}$ the equilibrium relation between output and profits is given by:*

$$\frac{1}{1-\alpha}d\mathbf{Y} = \underbrace{\alpha_L \mathbf{M}^Z d\mathbf{Y}}_{\text{Multiplier}} - \underbrace{\left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] d\mathbf{\Pi}}_{\text{Distributional channel}} - \underbrace{(1-\alpha_L)\,\mathbf{M}^Z d\mathbf{P}^*_{\mathbf{X}}}_{\text{Direct effect}} \tag{26}$$

Proof: appendix B.6.

Proposition (4) shows that even with an equal incidence of labor income and profits $\mathbf{M}^\Pi = \mathbf{M}^Z$, the cost-push shock has a contractionary effect on domestic output to the extent that MPCs are positive, $\mathbf{M}^\Pi = \mathbf{M}^Z > \mathbf{0}$. In this case the dynamics of the real wage and profits do not matter because both have the same effect on aggregate demand, and one can solve for the GE output response independently of the Philips curves:

Corollary 2. *If there is equal incidence of labor income and profits, $\mathbf{M}^\Pi = \mathbf{M}^Z$, then the output response to a foreign cost-push shock is:*

$$d\mathbf{Y} = -\left[\frac{\mathbf{I}}{(1-\alpha)\,\alpha_L} - \mathbf{M}^\Pi\right]^{-1} (1-\alpha_L)\,\mathbf{M}^\Pi d\mathbf{P}^*_{\mathbf{X}}$$

and thus contractionary if $\mathbf{M}^\Pi > \mathbf{0}$.

Corollary 2 suggests that any heterogeneous agent model with $\mathbf{M}^\Pi = \mathbf{M}^Z > 0$ will feature contractionary foreign cost-push shocks even with $\mathbf{M}^Z = \mathbf{M}^\Pi$, contrary to domestic markup shocks. Figure 7 illustrates this in the quantitative model. The leftmost panel shows that for the RA model there is no effect on output since $\mathbf{M}^Z = \mathbf{M}^\Pi = 0$, and the central bank keeps the real interest rate fixed. The middle panel shows that with positive MPCs but an equal profit/labor income incidence output drops but only modestly.[19]

[19] In the HANK model with equal incidence I calibrate the MPC out of labor income to 4% annually, which is the MPC out of profits in the baseline model, cf. Section 2.10.

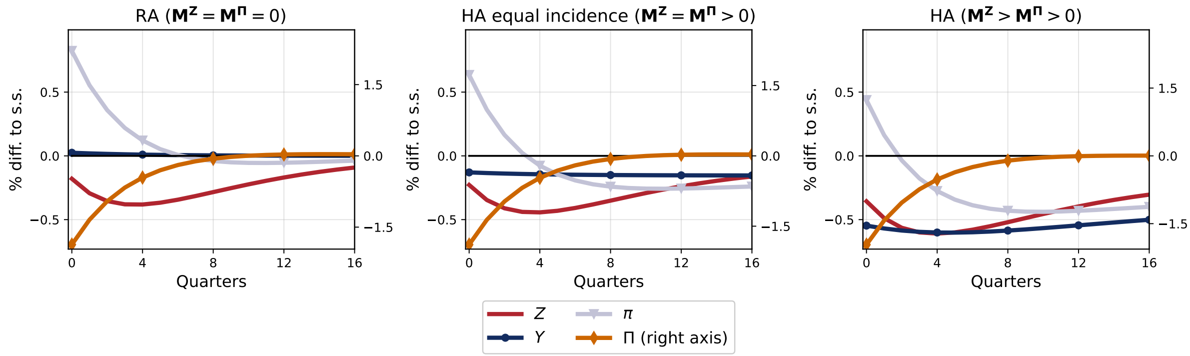


Figure 7: Effect of a cost-push shock in the RA and HA models

Note: Impulse responses to an AR(1) shock to a foreign cost-push shock with persistence 0.8. The initial increase in import prices is 10%.

For the more general case where $\mathbf{M}^Z \neq \mathbf{M}^\Pi$, the full general equilibrium solution for output is given in Proposition 5.

Proposition 5. *The equilibrium response of output $d\mathbf{Y}$ to a foreign cost-push shock is:*

$$d\mathbf{Y} = -\mathcal{M} \left\{ \underbrace{\mathbf{M}^\Pi}_{\text{Direct effect}} + \underbrace{\left(\mathbf{M}^Z - \mathbf{M}^\Pi\right) \boldsymbol{\Theta}^\mu}_{\text{Distributional channel}} \right\} (1 - \alpha_L)\, d\mathbf{P}^*_\mathbf{X} \tag{27}$$

where:

$$\mathcal{M} \equiv \left[((1-\alpha)\,\alpha_L)^{-1} \mathbf{I} - \mathbf{M}^\Pi - \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] \boldsymbol{\Theta}^L \right]^{-1}$$

is the general equilibrium multiplier of output.
Proof: appendix B.6.

Proposition 5 highlights that the effect of a cost-push shock is composed of a direct effect proportional to the aggregate MPC out of profits and a distributional term, which resembles the distributional term obtained for markup shocks in Proposition 2. The rightmost panel in Figure 7 shows that in the case where $\mathbf{M}^Z > \mathbf{M}^\Pi$ the cost-push shock generates a significant contraction in terms of output, similar to the case for the domestic markup shock. In particular, the distributional channel account for roughly 75% of the decline in output over the first year.

4.2 Price and wage flexibility

For the markup shock considered in Section 3 fully flexible wages was a sufficient condition to neutralize the effects of the shock, $d\mathbf{Y} = 0$, because wage flexibility eliminated the distributional channel. This logic does not necessarily carry over to the cost-push shock: While flexible wages stabilize real labor income by avoiding large fluctuations in real wages, it does so at the cost of a larger drop in profits. If $\mathbf{M}^{\Pi} > 0$ the domestic economy may exhibit a recession even under flexible wages because profits decline. In Proposition 3 I derive the analytical solution of the model to a cost-push shock under flexible wages:

Corollary 3. *The equilibrium response of output $d\mathbf{Y}$ to a foreign cost-push shock under flexible wages is:*

$$d\mathbf{Y} = -\left[\left((1-\alpha)\,\alpha_L\right)^{-1}\mathbf{I} - (1-\alpha_L\phi)\,\mathbf{M}^{\Pi} - \alpha_L\phi\mathbf{M}^{Z}\right]^{-1}\mathbf{M}^{\Pi}\,(1-\alpha_L)\,d\mathbf{P}^{*}_{\mathbf{X}} \tag{28}$$

Unlike the markup shock flexible wages are not sufficient to fully neutralize the effect of the cost-push shock on output because profits decline, and they affect aggregate demand by exactly $\mathbf{M}^{\Pi}$. Still, if $\mathbf{M}^{\Pi}$ has small initial entries the overall impact is smaller than the case with sticky wages, for the same reason laid out above: MPCs out of labor income tend to be larger than those out of profits.

For completeness Proposition 4 highlights the response under flexible prices. Compared to the response under flexible wages there is larger direct effect captured by the term $\alpha_L\mathbf{M}^{Z}$ – because real wages decline more initially – but a *smaller* general equilibrium multiplier. The smaller multiplier arises because markups are constant when prices are fully flexible, and so the feedback loop between the two Philips curves is shut down. Thus – unlike the markup shock – there is ambiguity as to whether the responses are smaller or larger with flexible prices vs. flexible wages.

Corollary 4. *The equilibrium response of output $d\mathbf{Y}$ to a foreign cost-push shock under flexible prices is:*

$$d\mathbf{Y} = -\left[\left((1-\alpha)\,\alpha_L\right)^{-1}\mathbf{I} - \mathbf{M}^{\Pi}\right]^{-1}\left\{(1-\alpha_L)\,\mathbf{M}^{\Pi} + \alpha_L\mathbf{M}^{Z}\right\}(1-\alpha_L)\,d\mathbf{P}^{*}_{\mathbf{X}} \tag{29}$$

Figure 8 compares the impulses in the baseline model with those under flexible wages and flexible prices respectively. Fully flexible prices amplify the output effects of the shock due to a large decline in real wages, as with the markup shock. With fully flexible wages the output effects are smaller, but more *persistent*, reflecting that the shock is concentrated on firm owners who have low, but persistent MPCs as seen in Figure 2c.[20] Note that the drop in consumption is larger than the direct "partial equilibrium" effect of profits on consumption $\mathbf{M}^{\Pi} \times d\Pi$ would suggest because the overall effect is scaled up by the general equilibrium multiplier, which is larger due to higher MPCs out of labor income. This is exactly the intuition behind eq. (28).

[20] The inflation response is negative in the case with flexible wages because the drop in demand is very persistent. With a fully forward looking Philips curve the future decline in demand dominates the initial increase in costs and firms reduce prices.

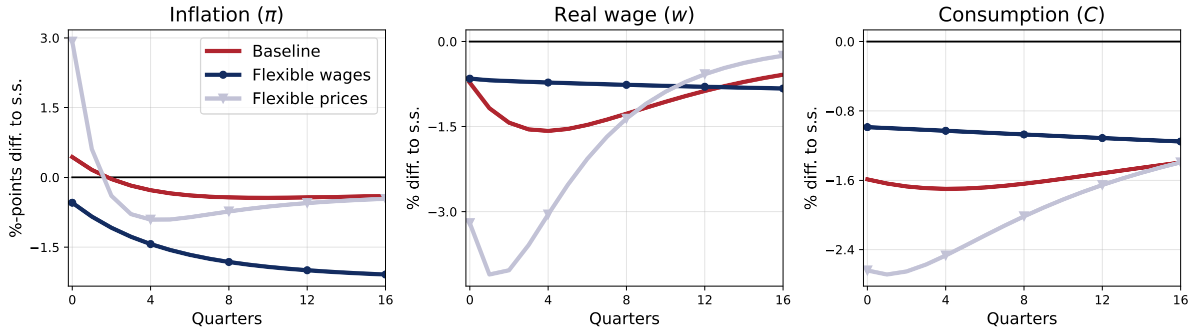


Figure 8: Cost-push shock and nominal rigidities

Note: Impulse responses to a foreign cost-push shock in HANK in the baseline model, with flexible wages, and flexible prices respectively.

4.3 Substitution in production

The analytical results assume zero substitution between production inputs, and the quantitative model assumes a relatively low elasticity of substitution between labor and intermediate goods ($\nu = 0.03$). Figure 9 shows how the transmission of the cost-push shock changes with the degree of substitutability. As the elasticity becomes larger firms respond to the increase in import prices by substituting away from materials towards labor, which is relatively cheap due to sticky nominal wages. This increases employment thereby stabilizing real labor income and consumption. Thus a necessary condition for the amplification of the cost-push shock is the presence of sticky wages *and* complementarity in factor inputs. A similar point was made in Lorenzoni and Werning 2023; Auclert et al. 2023a. Note, however, that an implausibly high degree of substitutability is required for consumption to *increase* in response to the shock.

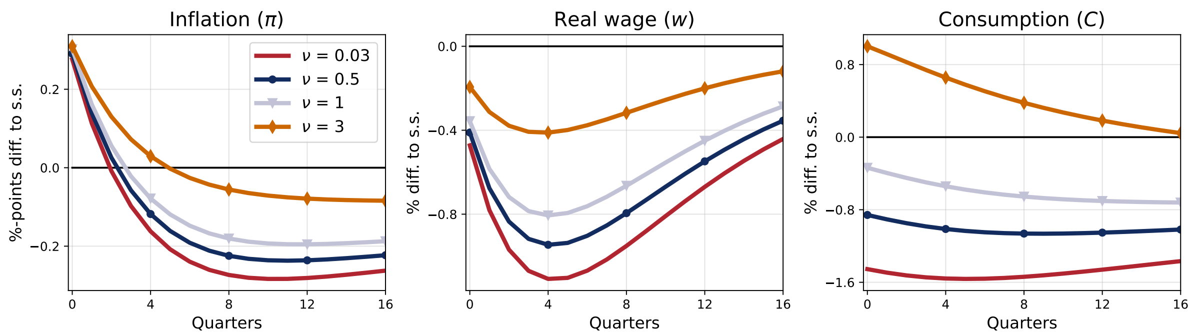


Figure 9: Importance of labor-materials substitution

Note: Impulse responses to a foreign cost-push shock in HANK with varying elasticity of substitution between labor and materials ν. The baseline features $\nu = 0.03$.

5 Optimal Monetary Policy and Foreign Cost-Push Shocks

Sections 3-4 analyzed the transmission of supply shocks, and show that they tend to be amplified in the presence of household heterogeneity. In this section I analyze what the *optimal monetary policy* response is to a foreign cost-push shock, and evaluate how it is shaped by heterogeneity and the open economy dimension of the model. The section constitutes a contribution to a recent burgeoning literature on optimal stabilization policy with heterogeneous agents, see for instance Bhandari et al. (2021), LeGrand and Ragot (2022), Davila and Schaab (2023), McKay and Wolf (2023) and Acharya et al. (2023). The first section 5.1 briefly presents a simple two-period model which is a special case of the full model in section 2, and discusses the forces shaping optimal monetary policy. The second part of the section formalizes the full Ramsey problem and solves it for the baseline open economy HANK model.

5.1 Stylized model of optimal policy

To understand the trade-offs faced by the social planner when choosing the optimal policy, I derive the first-order conditions in a stylized two-period model. The model is laid out in appendix C.1, but is briefly described here. The model is a special case of the general model presented in Section 2 with the following simplifying assumptions: Nominal wages are fixed, domestic households spend a fixed share on foreign and domestic goods, firms use variable labor and a fixed amount of foreign materials in production, and domestic firms are owned by foreign households.[21] All of these assumptions can easily be relaxed, but they aid in simplifying exposition. The economy is hit by aggregate shocks in period t, and the social planner chooses policy instruments optimally in this period as well. In period $t+1$ and onwards the economy returns to steady state as in Guerrieri et al. (2021). The aggregate shock is an increase in the price of imported materials $P^*_{X,t}$, as analyzed in Section 4.

5.1.1 Problem of the social planner

Denote by λ_t, ζ_t the Lagrange multipliers on the goods market clearing constraint and the UIP condition respectively. The optimal policy maximizes the following Lag-

[21] The assumption that firms are owned by foreigners implies $\mathbf{M}^{\Pi} = 0$ in the notation of Section 3. This assumption is not necessary for the results, but helps exposition.

rangian w.r.t $\{r_t, Y_t, Q_t, \lambda_t, \zeta_t\}$:

$$\begin{aligned}\mathcal{L}(\bullet) = &\int u\left(c_i\left(Z_t, r_t\right)\right) \mathrm{d}i - v\left(L_t\right) \\ &+ \lambda_t\left[\left(C^*_{H,t}\left(Q_t\right) + (1-\alpha)\int c_i\left(Z_t, r_t\right)\mathrm{d}i + \frac{\theta^P}{2}\pi^2_{H,t}Y_t\right) - Y_t\right] \\ &+ \zeta_t\left[(1+r^*) - (1+r_t)\,Q_t\right],\end{aligned}$$

where c_i is the consumption function of household i. Denoting by $m^Z_{it} \equiv \frac{\partial c_{it}}{\partial Z_t}, m^r_{it} \equiv \frac{\partial c_{it}}{\partial r_t}$ the change in individual consumption w.r.t real income and the interest rate, the first-order conditions are:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial Q_t} &= \int u'\left(c_{it}\right) m^Z_{it}\frac{\partial Z_t}{\partial Q_t}\mathrm{d}i + \lambda_t\frac{\partial C^*_{H,t}}{\partial Q_t} + \lambda_t(1-\alpha)\frac{\partial C_t}{\partial Z_t}\frac{\partial Z_t}{\partial Q_t} + \lambda_t\theta^P\pi_{H,t}Y_t\frac{\partial \pi_{H,t}}{\partial Q_t} - \zeta_t(1+r_t) = 0, \\ \frac{\partial \mathcal{L}}{\partial Y_t} &= \int u'\left(c_{it}\right) m^Z_{it}\frac{\partial Z_t}{\partial Y_t}\mathrm{d}i - v'\left(L_t\right)\frac{\partial L_t}{\partial Y_t} - \lambda_t\left[1-(1-\alpha)\frac{\partial C_t}{\partial Z_t}\frac{\partial Z_t}{\partial Y_t} - \theta^P\pi_{H,t}\frac{\partial \pi_{H,t}}{\partial Y_t} - \frac{\theta^P}{2}\pi^2_{H,t}\right] = 0, \\ \frac{\partial \mathcal{L}}{\partial r_t} &= \int u'\left(c_{it}\right) m^r_{it}\,\mathrm{d}i + \lambda_t(1-\alpha)\frac{\partial C_t}{\partial r_t}di - \zeta_t Q_t = 0.\end{aligned}$$

To understand the intuition for these, I consider a case where the central planner has access to capital controls (for instance through a tax on foreign bond holdings) as in Farhi and Werning (2014). This implies that the UIP condition is not binding, $\zeta_t = 0$, and the planner can effectively choose r and Q separately as controls. The first-order condition for optimal interest rate setting then reads:

$$\underbrace{\int u'\left(c_{it}\right) m^r_{it}di}_{\text{Loss from higher rate}} = \underbrace{-\lambda_t(1-\alpha)\frac{\partial C_t}{\partial r_t}}_{\text{Gain/loss from low demand}} \tag{30}$$

The left-hand side term is the direct effect from a higher interest rate on consumption. This effect tends to be positive for savers and negative for borrowers, and the aggregate effect is therefore ambiguous, see Nuño and Thomas (2022). The right-hand side is the loss ($\lambda_t < 0$) or gain ($\lambda_t > 0$) coming from lower demand due to intertemporal substitution effect on consumption. Using the first-order condition for Y_t the multiplier λ_t can be expressed as:

$$\lambda_t = \mathcal{A}_t\left[\int u'\left(c_{it}\right) m^Z_{it} w_t\frac{\partial L_t}{\partial Y_t}di - v'\left(L_t\right)\frac{\partial L_t}{\partial Y_t}\right],$$

where $\mathcal{A}_t^{-1} = 1-(1-\alpha)\frac{\partial C_t}{\partial Z_t}\frac{\partial Z_t}{\partial Y_t} - \theta^P\pi_{H,t}\frac{\partial \pi_{H,t}}{\partial Y_t} - \frac{\theta^P}{2}\pi^2_{H,t}$. λ_t captures the social value of an increase in domestic output Y_t: An increase in output requires more use of labor in production and therefore increases real income, the individual value of which is

$u'(c_{it})\, m^Z_{it}$. Still, an increase in labor supply also generates disutility of working, captures by $\nu'(L_t)\, \frac{\partial L_t}{\partial Y_t}$. In the quantitative model below I find that λ_t tends to be positive in the HANK model (reflecting the social value of higher real income) and negative in RANK (reflecting primarily the disutility of labor). The first-order condition for the real exchange rate when the UIP constraint is slack ($\zeta_t = 0$) reads:

$$\underbrace{\int u'(c_{it})\, m^Z_{it} L_t \frac{\partial w_t}{\partial Q_t} di}_{\text{Gain from higher real wages}} + \underbrace{\lambda_t \theta^P \pi_{H,t} Y_t \frac{\partial \pi_{H,t}}{\partial Q_t}}_{\text{Gain from lower inf.}} + \underbrace{\lambda_t (1-\alpha) \frac{\partial C_t}{\partial Z_t} \frac{\partial w_t}{\partial Q_t} L_t}_{\text{Keynesian multiplier}} = \underbrace{-\lambda_t \frac{\partial C^*_{H,t}}{\partial Q_t}}_{\text{Loss from exp. switching}} \tag{31}$$

The first term represent the gain from generating an appreciation of the real exchange rate. An appreciation reduces the price of imports in domestic currency, thereby lowering domestic producer prices. This reduces CPI inflation, thereby increasing real wages. To see this more clearly, the derivative $\frac{\partial w_t}{\partial Q_t}$ can be written:

$$\frac{\partial w_t}{\partial Q_t} = -\frac{W}{P_t^2} \left[\underbrace{\alpha \frac{\partial P_{F,t}}{\partial Q_t}}_{\text{Final goods import price}} + (1-\alpha) \underbrace{\frac{\partial P_{H,t}}{\partial P^*_{X,t}} \times \frac{\partial P^*_{X,t}}{\partial Q_t}}_{\text{Domestic prices}} \right].$$

The first term in the brackets captures the effect of an exchange rate appreciation on the import prices of final goods which enter the consumption basket of domestic households directly. The second term is the indirect effect on domestic producer prices: An appreciation reduces import prices of materials by $\frac{\partial P^*_{X,t}}{\partial Q_t}$. This reduces marginal costs and therefore producer prices by $\frac{\partial P_{H,t}}{\partial P^*_{X,t}}$. The size of this pass-through depends on the slope of the Philips-curve. The central insight here is that the strength of the real wage/exchange rate channel depends on the pass-through of exchange rates to import prices, $\frac{\partial P_{F,t}}{\partial Q_t}, \frac{\partial P^*_{X,t}}{\partial Q_t}$.

The second term in (31) captures the welfare effect induced by a lower resource loss to the extent that an exchange rate appreciation reduces domestic inflation. The third term is a general equilibrium effect. The real wage gain from an appreciation translates into higher real income, which increases aggregate demand. The welfare gain (or loss) from this is measured by the multiplier λ_t. The final term in (31) relates to *expenditure switching*. When the exchange rate appreciates foreign consumers substitute towards cheaper goods in other countries, thus reducing the demand for domestic goods. In HANK this constitutes a welfare loss since the social value of output is positive, $\lambda_t > 0$.

In the general model below there are no capital controls and so the UIP condition generally binds ($\zeta_t \neq 0$). This implies that, in addition to the trade-offs explained above, the gains and losses from a higher real interest rate must also take into account gains and losses from exchange rate movements.

5.1.2 Role of heterogeneity and MPCs

In this section I briefly explain how heterogeneity and incomplete markets shape the gains and losses faced by the social planner. Starting from the first term in eq. (31), an appreciation generates a real income gain of $dZ_t = L_t \frac{\partial w_t}{\partial Q_t}$. At the individual level the increase in consumption arising from this income gain is m_{it}^Z, which increases individual utility by the marginal utility of consumption $u'(c_i)$. To see how heterogeneity matters for the optimal policy, note that the marginal social welfare gain from this increase in real income can be decomposed as:[22]

$$\int u'(c_{it})\, m_{it}^Z \,\mathrm{d}i \times dZ_t \approx \left[u'(C_t) \frac{\partial C_t}{\partial Z_t} + \frac{1}{C_t} \mathbb{V}\mathrm{ar}(c_{it}) \frac{\partial C_t}{\partial Z_t} + \mathbb{C}\mathrm{ov}\left(u'(c_{it}), m_{it}\right) \right] \times dZ_t \tag{32}$$

First, with complete markets households are able to almost completely smooth transitory shocks implying $\frac{\partial C_t}{\partial Z_t} = \frac{\partial c_{it}}{\partial Z_t} \approx 0$, so the marginal welfare gain from an increase in real income is zero. With positive MPCs ($\frac{\partial C_t}{\partial Z_t} > 0, \frac{\partial c_{it}}{\partial Z_t} > 0$), changes in real income affect aggregate welfare. The decomposition shows that this gain comes from three distinct sources. The first term reflects the average consumption gain in the economy. The second term shows that the gains are larger with heterogeneity (since $\mathbb{V}\mathrm{ar}(c_{it}) > 0$). This is due to the concavity of the utility function, which implies that a uniform increase in consumption across all households generates larger welfare gains when there is initial consumption dispersion in the economy. The final term captures that an increase in aggregate real income has additional welfare gains if households with high marginal utility of consumption also exhibit higher MPCs (implying $\mathbb{C}\mathrm{ov}\left(u'(c_{it}), \frac{\partial c_{it}}{\partial Z_t}\right) > 0$). In the present model that is the case since poorer households - who consume less - also tend to be closer to the borrowing constraint.

As noted above the standard New Keynesian model contains none of the terms in (32). TANK models without consumption inequality such as Chan et al. (2024) contain only the first term. Models with heterogeneity but without behavioral differences[23] – as in Blanchard-Yaari perpetual youth type models (Farhi and Werning (2019), Acharya et al. (2023) and Angeletos et al. (2024)) – feature terms 1) and 2) but not 3) since all households have the same MPC. Standard incomplete market models such as Bewley (1986), Imrohoroğlu (1989), Huggett (1993) and Aiyagari (1994) generally feature all

[22] See appendix C.3 for derivations, and Bhandari et al. (2023), Dávila and Schaab (2023) for related decompositions.

[23] That is, models which feature linear policy functions.

three channels.

5.2 The Ramsey problem in the quantitative model

I now turn to the quantitative model laid out in Section 2. I am interested in computing the constrained-efficient allocation. That is, the overall aim is to calculate the monetary policy response (cast in terms of interest rates) which maximizes aggregate welfare subject to implementability constraints. Here implementability constraints captures the notion that the social planner must respect the optimizing behavior of households and firms when choosing the optimal sequence of interest rates.

I assume perfect-foresight w.r.t aggregates and formulate the Ramsey problem in sequence-space. To fix notation, let $\boldsymbol{X} = (\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots)'$ denote endogenous variables of the model, where $\boldsymbol{x}_t$ is an $n_x \times 1$ vector and n_x is the number of endogenous variables in the model. Similarly, let $\boldsymbol{Z}$ denote policy instruments, and $\boldsymbol{\epsilon}$ denote aggregate (MIT) shocks. Using a sequence space-representation we can write the equilibrium of the model as a system of non-linear equations:

$$\mathcal{H}(\boldsymbol{Z}, \boldsymbol{X}, \boldsymbol{\epsilon}) = 0,$$

where $\mathcal{H}$ contains the residuals of the model equations (1)-(17). Following Aiyagari (1995) aggregate welfare is defined as the ex-ante utilitarian welfare function:[24]

$$\mathcal{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \left[u\left(c_t(e,a)\right) - v\left(L_t\right)\right] d\mathcal{G}_0(e,a), \tag{33}$$

where $\mathbb{E}_0$ is the expectation at time 0 w.r.t idiosyncratic uncertainty. Note here that even though there is no aggregate uncertainty, uncertainty at the micro level is fully incorporated in the Ramsey problem. The optimal Ramsey policy solves the following maximization problem:

$$\max_{\{\boldsymbol{Z}, \boldsymbol{X}\}} \mathcal{W} \quad \text{s.t} \quad \mathcal{H}(\boldsymbol{Z}, \boldsymbol{X}, \boldsymbol{\epsilon}) = 0. \tag{34}$$

Let $\boldsymbol{M}$ denote the vector of Lagrange multipliers associated with the residuals $\mathcal{H}(\bullet)$. The Lagrangian associated with the Ramsey problem is:[25]

$$\mathcal{L}(\bullet) = \mathcal{W} + \boldsymbol{M}'\mathcal{H}(\boldsymbol{Z}, \boldsymbol{X}, \boldsymbol{\epsilon}). \tag{35}$$

[24]The solution procedure can easily incorporate deviations from utilitarian welfare functions.

[25]I focus on the Ramsey problem cast in dual form, where the social planner explicitly chooses the optimal level of the instrument. This is opposed to the primal form of the problem used in the optimal taxation literature (e.g. Chari and Kehoe 1999), where one first computes the efficient allocation among implementable allocations, and afterwards backs out the instruments which implements that allocation.

Letting $\nabla_{\boldsymbol{Z}}, \nabla_{\boldsymbol{X}}$ denote gradients w.r.t $\boldsymbol{Z}, \boldsymbol{X}$, the first-order conditions of the social planner are:

$$\left[\begin{array}{c} \nabla_{\boldsymbol{Z}}\mathcal{W} + \nabla_{\boldsymbol{Z}}\boldsymbol{M}'\mathcal{H}\left(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{\epsilon}\right) \\ \nabla_{\boldsymbol{X}}\mathcal{W} + \nabla_{\boldsymbol{X}}\boldsymbol{M}'\mathcal{H}\left(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{\epsilon}\right) \\ \mathcal{H}\left(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{\epsilon}\right) \end{array}\right] = \boldsymbol{0}, \tag{36}$$

To solve the optimal policy problem numerically, I compute the gradients using automation differentiation.[26] I solve the households' problem using the endogenous grid method (Carroll 2006), and approximate the endogenous distribution $\mathcal{G}$ using the histogram method of Young (2010). Both of these methods are well suited for automatic differentiation, unlike the more conventional methods such as value function iteration for the household problem, and stochastic Monte Carlo simulation methods for the simulation. Given the gradients I proceed and solve the system (36) using a quasi-Newton solver, supplied with the Jacobian of (36), again computed using automation differentiation.

Constrained optimal steady state. I first compute the long-run optimal steady state w.r.t the instruments available to the planner, i.e. the nominal interest rate. As is well known in the literature, the optimal steady state of a planner that only has the interest rate as instrument coincides with the zero inflation competitive steady state.[27] The reason is that monetary policy cannot affect the real interest rate in the steady state (this is determined from abroad by the UIP condition eq. (15)) and cannot generate surprise inflation to remedy market inefficiencies, so the only effect of a higher nominal rate i is to drive up steady state inflation π through the Fisher equation, which generates an inefficient resource loss in the form of the Rotemberg adjustment cost. Therefore the constrained efficient steady state features $i = i^* = r^*$ and $\pi = 0$.

Time-inconsistency. Even though the long run steady state is (constrained) efficient, the combination of inefficiencies and forward-looking equations (households' Euler equations, the UIP condition etc.) imply that optimal monetary policy is time-inconsistent, meaning that given the opportunity the social planner will engineer unexpected policy shocks even in the absence of aggregate shocks in order to temporarily reduce inefficiencies and raise aggregate welfare. In order to isolate the part of the optimal Ramsey plan that is concerned with the stabilization of aggregate shocks I focus on a timeless approach as formulated by Woodford (2003) – sometimes referred to as the optimal Ramsey policy under *commitment*. In practice this amounts to fixing the initial values

[26] I use Google JAX in Python for this purpose.

[27] See e.g. González et al. (2022) for a closed economy or Faia and Monacelli (2008) for an open economy.

of Lagrange multipliers on forward-looking equations in eq. (36) to their steady-state values, see appendix C.4 for details.

Related methods. A number of different methods have been applied in the recent literature to compute optimal policies in models with heterogeneous agents. I here provide a brief overview of the methods, and relate them to my solution procedure. Bhandari et al. (2021) compute optimal policy in the state-space using numerical perturbation methods. The main limitation of their method is that it does not handle occasionally binding borrowing constraints or non-linearities at the micro level. The perfect-foresight solution I compute in the sequence-space can handle both of these features. LeGrand and Ragot (2022) compute optimal policies in the state-space by deriving the FOCs of the planner by hand, and then solving these numerically using Dynare. In the standard discrete time incomplete markets model deriving the FOCs of the planner is infeasible because it involves differentiating the endogenous distribution $\mathcal{G}$. They circumvent this issue by using a truncation approach where each household is described by their history of idiosyncratic states, truncated at some past period. This implies a finite state-space representation, which makes deriving analytical first-order conditions feasible. Davila and Schaab (2023) use a similar procedure, but avoid truncation by working in continuous time, where the law of motion of the endogenous distribution of households has a functional form. The closest related method to mine is González et al. (2024) who study a heterogeneous firm environment. They work in continuous time, but compute the FOCs of the planner problem using symbolic differentiation. McKay and Wolf (2023) follow the approach in Woodford (2003). They derive a second-order approximation of the aggregate welfare function around the efficient steady state, and can therefore use standard linearization techniques to characterize the optimal policy. The steady state of the HANK model in the present paper is generally inefficient because of various distortions (incomplete markets, firm and union market power, terms of trade externalities etc.), and the Ramsey problem can therefore not be cast in a linear-quadratic form.

5.3 Additional model elements

Before proceeding with the quantitative model, I extend the model in two dimensions which the first-order condition (31) highlighted as being important for optimal policy in a small open economy.

Expenditure switching. I extend the model with dynamic adjustment of consumption baskets to match the degree of expenditure switching observed in the data. As before total consumption of goods C_t is a CES aggregate over foreign and domestic goods with elasticity of substitution η:

$$C_t = \left[\alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

I introduce dynamic trade elasticites into the model through delayed substitution using a Calvo-type mechanism (Auclert et al. 2024b). Let $x_{H,t} = \frac{P_{H,t}C_{H,t}}{P_t C_t}$ be the share of spending on domestic goods in units of the CPI. With probability $1-\theta^C$ households may adjust the share of consumption going to domestic vs. foreign goods, and with probability θ^C they are forced to keep the ratio constant until the next period. This problem leads to the following dynamic, implicit equation for the target ratio $x_{H,t}$:

$$\frac{\mathring{x}_{H,t}}{1-\mathring{x}_{H,t}} = \left[\frac{\sum_{s=0}^{\infty}\left(\beta\theta^C\right)^s \alpha^{\frac{1}{\eta}}\left(\frac{C_{t+s}}{p_{F,t+s}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{s=0}^{\infty}\left(\beta\theta^C\right)^s (1-\alpha)^{\frac{1}{\eta}}\left(\frac{C_{t+s}}{p_{H,t+s}}\right)^{\frac{\eta-1}{\eta}}}\right]^{-\eta}, \tag{37}$$

$$x_{H,t} = \left(1-\theta^C\right)\mathring{x}_{H,t} + \theta^C x_{H,t-1}, \tag{38}$$

where $\mathring{x}_{H,t}$ is the targeted ratio of $x_{H,t}$ when allowed to re-optimize (see appendix C.2 for details and the full solution). Given $x_{H,t}$ one can calculate $C_{H,t}$ from the definition of $x_{H,t}$. $C_{F,t}$ can then be calculated from the constraint $P_t C_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. Note that with $\theta^C = 0$ one recovers the usual CES demand functions. Foreign demand for domestic goods $C^*_{H,t}$ is modeled similar to domestic demand implying:

$$\frac{\mathring{x}^*_{H,t}}{1-\mathring{x}^*_{H,t}} = \left[\sum_{s=0}^{\infty}\left(\beta\theta^C\right)^s (\alpha^*)^{\frac{1}{\eta}}\left(\frac{C^*_{t+s}}{p^*_{H,t+s}}\right)^{\frac{\eta-1}{\eta}}\right]^{\eta}, \tag{39}$$

$$x^*_{H,t} = \left(1-\theta^C\right)\mathring{x}^*_{H,t} + \theta^C x^*_{H,t-1}. \tag{40}$$

I calibrate the reset probability $1-\theta^C$ to match the response of trade following a permanent shock to tariffs found in Boehm et al. (2023), see Figure 10. I first fit a persistent AR(1) process which I assume to stabilize at a permanent level after 10 years, see the left panel in Figure 10. Afterwards I feed this fitted shock into the delayed substitution model and minimize the distance between the model response and the empirical evidence using the reset probability $1-\theta^C$. This delivers $\theta^C = 0.944$, and provides a good fit to the empirical evidence on dynamic trade elasticities.

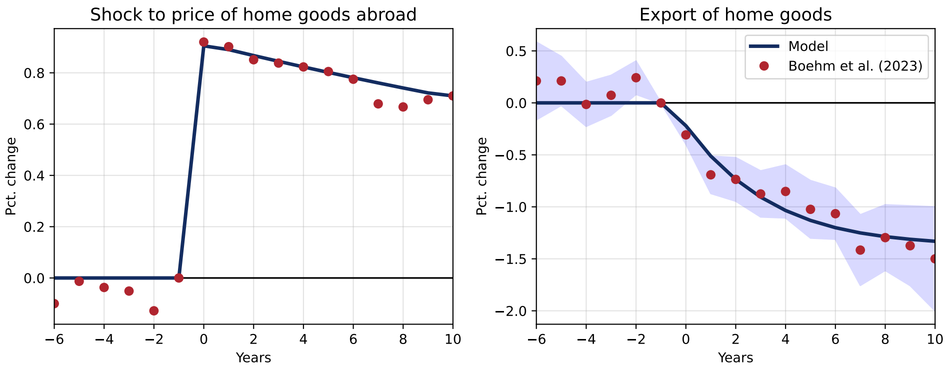


Figure 10: Calibration of short-run trade elasticity

Note: The Figure shows the tariff shock and estimated response of trade flows following Boehm et al. (2023) against the calibrated model.

Incomplete exchange rate pass-through. The second extension I introduce is a deviation from the law of one price by allowing for incomplete *exchange rate pass-through* (EPRT) into domestic import prices. I do this in a simple way where I assume that firms and households only update the exchange rate, they apply to imported goods, with probability $\theta^{\mathcal{E}}$, and otherwise use the exchange rate from when they last updated. This implies the following evolution of the domestic import price for final goods and imported materials respectively:

$$P_{F,t} = P^*_{F,t}\theta^{\mathcal{E}} \sum_{s=0}^{\infty} \left(1-\theta^{\mathcal{E}}\right)^s \mathcal{E}_{t-s}, \tag{41}$$

$$P_{XF,t} = P^*_{X,t}\theta^{\mathcal{E}} \sum_{s=0}^{\infty} \left(1-\theta^{\mathcal{E}}\right)^s \mathcal{E}_{t-s}. \tag{42}$$

Note that in this simple framework the law of one price holds in the long run.[28] To calibrate the short-run pass-through I rely on the estimates from Campa and Goldberg (2005) presented in Figure 11. Their estimates suggest large cross-country heterogeneity in the ERPT, and are roughly consistent with the newer estimates in Burstein and Gopinath (2014). In the baseline model I calibrate the ERPT to the median estimate in Campa and Goldberg (2005), resulting in $\theta = 0.59$ such that the contemporaneous pass-through from a 1% change in the nominal exchange rate is 0.59%.

[28]This formulation also features full pass-through from foreign export prices $P^*_{F,t}, P^*_{X,t}$ into domestic import prices $P_{F,t}, P_{XF,t}$. I opt for this formulation since otherwise a change in the pass-through $\theta^{\mathcal{E}}$ would also affect directly the transmission of the shock, making it impossible to evaluate the role of the ERPT in shaping the optimal policy.

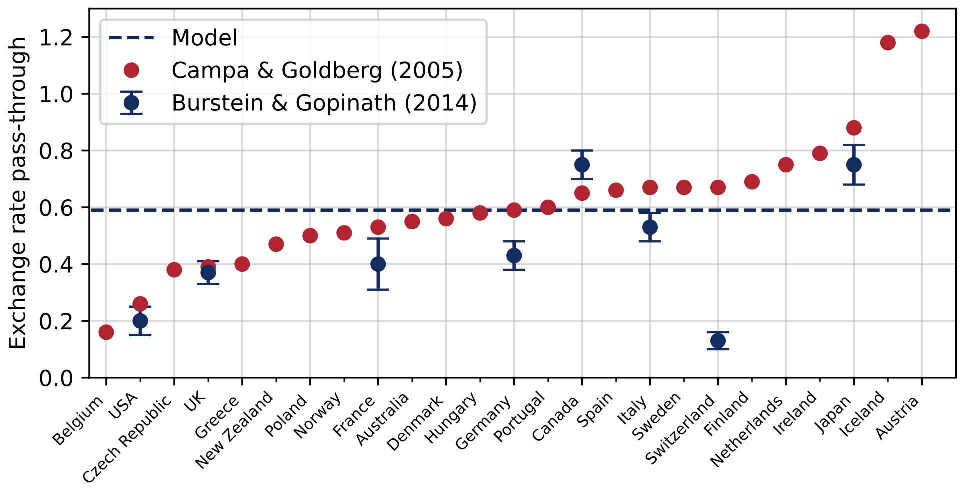


Figure 11: ERPT across countries

Note: Estimates of contemporaneous exchange rate pass-through to import prices from Campa and Goldberg (2005) (red) and Burstein and Gopinath (2014) (blue).

5.4 Results

This section presents the main results. I compute the optimal Ramsey monetary policy in the HANK model in response to a cost-push shock, and compare with the results obtained under a representative agent. The shock is the same as in the previous section, an AR(1) shock to the import prices of materials with persistence 0.8. The initial increase in import prices is 10%.

5.4.1 Closed economy

For comparability with the existing literature, Figure 12 presents the responses to a persistent cost-push shock under the optimal monetary policy in a closed economy. The closed economy setting is achieved by setting $\alpha = 0$ and by replacing the real UIP condition eq. (15) with the condition $Q_t = Q_{ss}$.[29] The optimal monetary policy response is less aggressive in HANK since the social planner has a larger incentive to stabilize employment and labor income due to the presence of poor households, resulting in moderate demand stabilization at the cost of higher inflation. This is in line with the results in Bhandari et al. (2021) and Smirnov (2022) for markup shocks and Davila and Schaab (2023) for demand shocks. Bilbiie (2024) show in a tractable closed economy HANK model that the social planner should attach more weight to the output-gap than inflation, implying more accommodating policy in general. Davila

[29] In formal terms the economy is not closed as firms still imports materials from abroad, which is the source of shock. With a constant real exchange rate the price of these imports is constant, so this can be interpreted as a shock to the endowment of domestic materials instead.

and Schaab (2023) and McKay and Wolf (2023) find roughly the same policy in HANK and RANK for cost-push shocks.

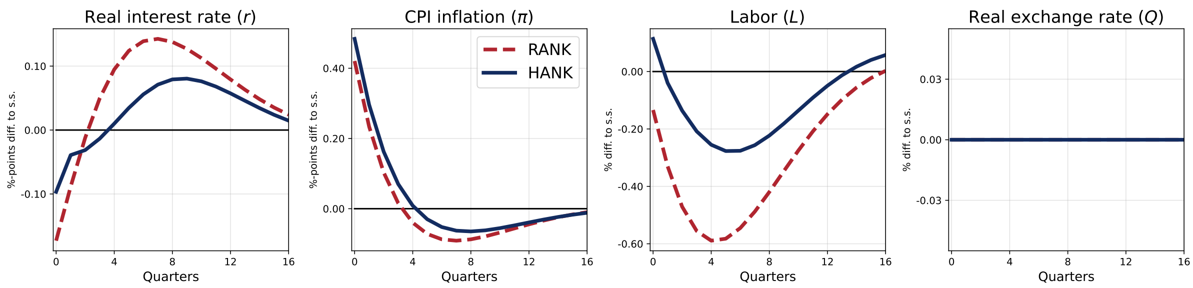


Figure 12: Optimal policy in the closed economy

5.4.2 Open economy

I now turn to the optimal monetary policy in the baseline open economy model. As seen in Figure 13 the optimal response entails a more accommodating policy in RANK, whereas the monetary policy response in HANK is more aggressive. The larger increase in interest rates implies a larger reduction in demand as well as a larger appreciation of the real exchange rate, both of which aids in bringing down inflation significantly compared to the optimal outcome in the RANK model. As the next two experiments showcase, the short run trade elasticity as well as a positive exchange rate pass-through are key for this outcome in the HANK model.

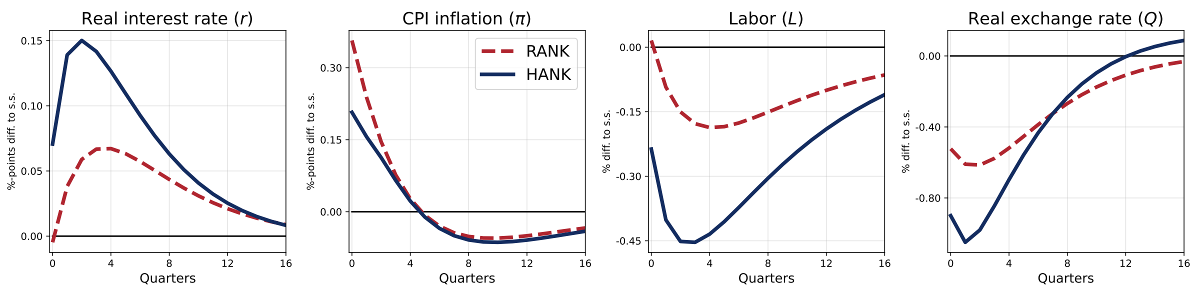


Figure 13: Optimal monetary policy response to a cost-push shock

Expenditure switching. In Figure 14 I recompute the optimal policy in the open economy model with varying levels of the (short-run) trade-elasticity by varying the stickiness θ^C in eq. (37)-(38), (39)-(40). Recall that with $\theta^C = 0$ the elasticity is $\eta = 2$ in the baseline calibration, and declines for larger θ^C. In HANK (first row of the Figure) the optimal monetary policy with a low short-run trade elasticity constitutes a large increase in the real interest rate as in Figure 13. Through the real UIP condition, this implies a larger appreciation of the real exchange rate. This alleviates some of the domestic price pressures since this lowers import prices faced by domestic consumers and firms, thereby increasing real labor income, as highlighted in eq. (31). With a higher trade elasticity this is not feasible for the social planner, since a larger appreciation of the real exchange rate will lead to expenditure switching. This implies a drop in exports and imports since domestic goods become more expensive and foreign goods cheaper, thereby reducing domestic employment and counteracting the stabilization of real labor income. This highlights that the optimal policy in HANK effectively leverages the real income channels of exchange rates highlighted in Auclert et al. (2024b) when the short-run degree of expenditure switching is low. In RANK (second row of the Figure) the optimal policy also results in a higher real interest rate under low trade elasticities, but the differences are much less stark. With a higher trade elasticity both the HANK and RANK model features mildly hawkish policy of similar magnitude. With a sufficiently high trade elasticity ($\eta \to \infty$) it is optimal to keep the real rate constant in both models and the optimal policy (but not the allocation) coincide.

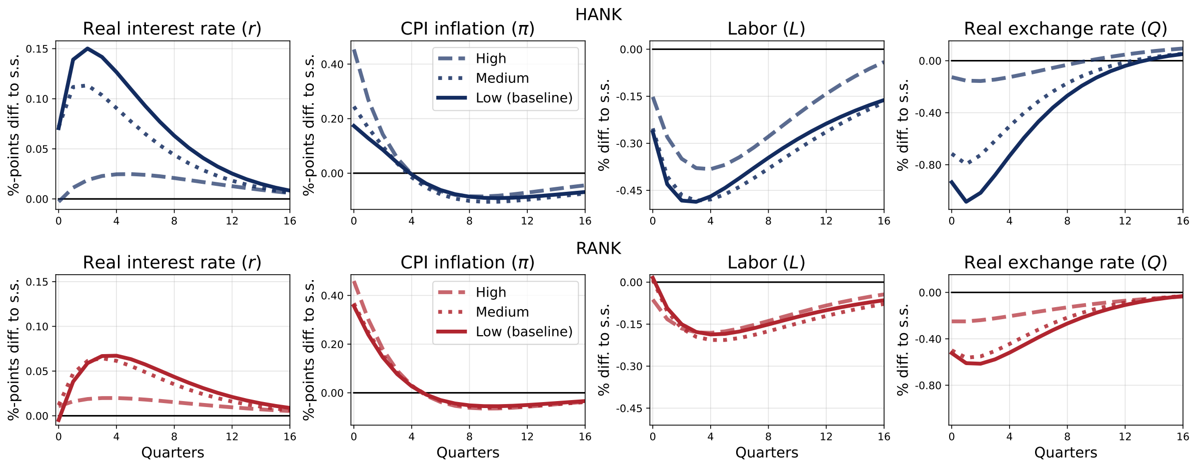


Figure 14: Optimal monetary policy response to a cost-push shock with varying trade elasticities η

Note: The Figure shows the responses under the Ramsey plan in HANK and RANK for a varying short-run trade elasticity. "Low (baseline)" corresponds to $\theta^C = 0.944$, "Medium" to $\theta^C = 0.7$, and "High" to $\theta^C = 0$.

Exchange rate pass-through. Equation (31) showed that in the HANK model the social planner has an incentive to appreciate the exchange rate in order to reduce import prices, thereby lowering domestic inflation and stabilizing real wages. Figure 15 shows the optimal monetary policy responses, varying the degree of exchange rate pass-through (ERPT) in the model by varying $\theta^{\mathcal{E}}$ in eqs. (41)-(42). In the HANK model the degree of ERPT to import prices is central to the overall magnitude of the interest rate response. When the pass-through is large, it is optimal for the planner to raise the interest rate significantly in order to appreciate the exchange rate which reduces import prices and raises real labor income. With a low pass-through the planner cannot affect import prices through exchange rate manipulation, and it is instead optimal to engineer a relatively weak interest rate response in order to balance the inflation-employment trade-off. This stands in contrast to the RANK model where the overall response of the interest rate is much less sensitive to the degree of exchange rate pass-through.

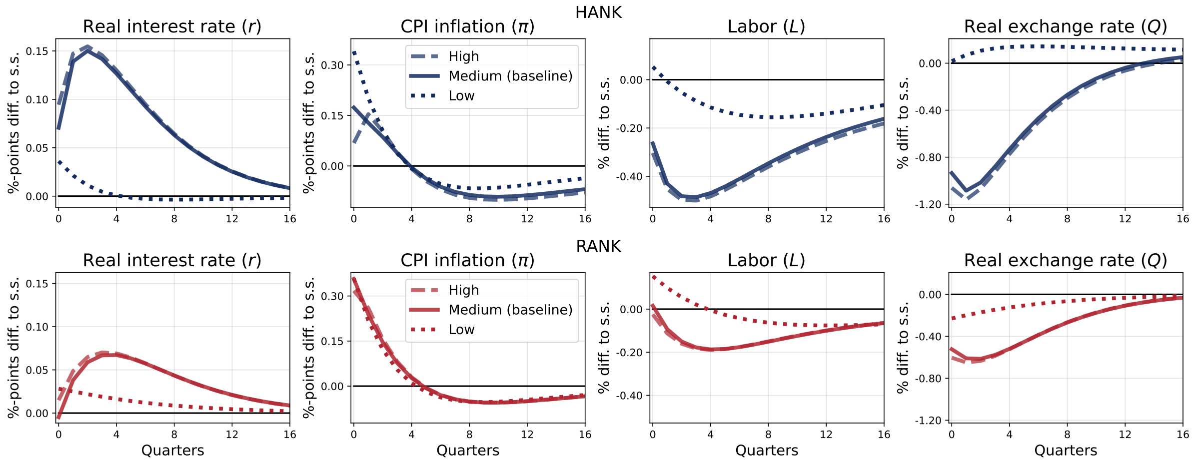


Figure 15: Optimal monetary policy response to a cost-push shock with varying ERPT to import prices

Note: The Figure shows the responses under the Ramsey plan in HANK and RANK for a varying degree of exchange rate pass-through. "Low" corresponds to $\theta^{\mathcal{E}} = 0.05$, "Medium (baseline)" to $\theta^{\mathcal{E}} = 0.59$, and "High" to $\theta^{\mathcal{E}} = 1$.

5.4.3 Extensions and robustness

Balance sheet effects. De Ferra et al. (2020) showed that when households are leveraged in foreign currency this may amplify the effect of exchange rate movements. To implement this effect in my baseline model I introduce gross debt into the model. I consider two scenarios: 1) The steady state NFA is negative, and all households are equally leveraged in foreign currency, 2) The steady state NFA is negative, but foreign currency debt is concentrated on poorer households, see Appendix C.5 for details. De Ferra et al. (2020) find that the latter amplifies the effect of capital flows even more since it produces a positive covariance between exchange rate exposure and MPCs. For both scenarios I set the steady state supply of foreign credit to 25% of total net wealth ($\frac{NFA}{A} = -0.25$), and for the second scenario I calibrate the model such that the average gross debt of households with zero net wealth is 24% of average household yearly labor income following De Ferra et al. (2020).

Figure 16 displays the results. A negative initial NFA implies that it is optimal to raise interest rates more since the exchange rate appreciating associated with higher rates reduces the value of outstanding foreign debt held by domestic households.[30]

[30]The opposite is the case when the initial NFA is positive.

When foreign currency debt is concentrated more on poor households (who exhibit both higher MPCs and a higher marginal utility of consumption) this channel gets amplified and the optimal response is even more hawkish.

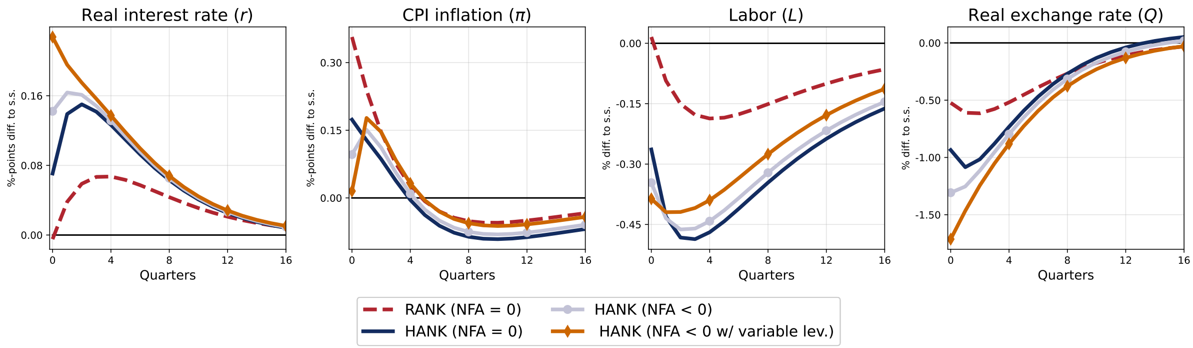


Figure 16: Optimal monetary policy response to a cost-push shock with balance sheet effects (HANK only)

Cyclical inequality. In this scenario I allow for cyclical inequality following the plethora of empirical evidence showing that earnings are more volatile over the business cycle at the bottom of the income distribution than at the top, and that the cross-sectional dispersion of earnings rises in recessions and decreases in expansions (Guvenen et al. (2017) and Storesletten et al. (2004)). Thus I follow Auclert and Rognlie (2018) and assume that real labor income of individual i is given by:

$$z_{it} = \frac{Z_t e_i^{1+\xi \ln \frac{Z_t}{Z}}}{\int e_i^{1+\xi \ln \frac{Z_t}{Z}} di}$$

where the elasticity ξ determines the cyclicality of earnings dispersion w.r.t aggregate labor income. I follow Acharya et al. (2023) regarding the calibration of ξ, and obtain $\xi < 0$ corresponding to counter-cyclical earnings risk.[31] Figure 17 plots the response under the optimal policy with counter-cyclical risk. Compared to the baseline HANK model with constant risk, it is optimal to raise interest rates even further initially, such that the initial inflation response is negative. This occurs due to a large currency appreciation. The reason for this is that endogenous cyclical inequality amplifies the social value of an increase in real labor income by increasing all terms in eq. (32). The aggregate MPC out of labor income increases because the labor income of poor, high MPC households now move more than 1-for-1 with aggregate labor income. Additionally, the covariance terms in (32) also increases since these households also tend to a higher marginal utility of consumption. Thus the presence of endogenous risk further strengthens the finding of a more hawkish monetary policy in HANK.

[31] Appendix C.6 presents the implementation details.

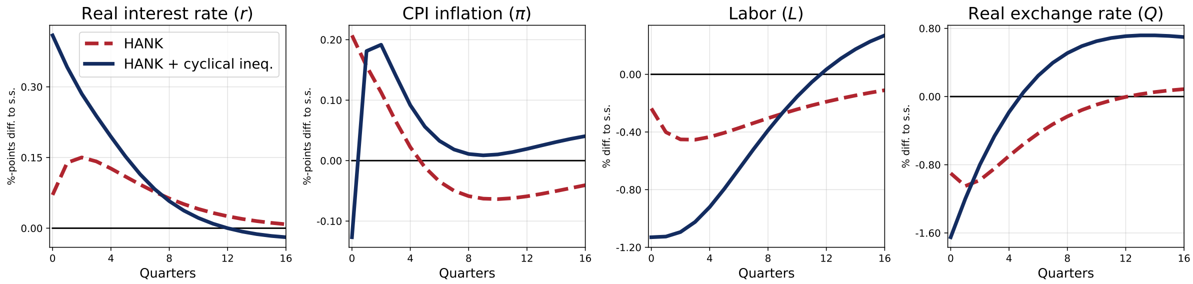


Figure 17: Optimal monetary policy response to a cost-push shock with cyclical inequality

5.4.4 Implications for welfare

As a last exercise I consider the implications of using the optimal monetary policy response implied by the RANK model in the HANK model.

Figure 18 plots the welfare loss experienced by households across the wealth distribution in consumption-equivalent units. That is, the Figure shows how much of annual steady-state consumption households are willing to give up in order to avoid the aggregate shock.[32] I plot the welfare effects in the HANK model for three different policies: 1) The optimal monetary policy response computed in the baseline HANK model, 2) A counterfactual where I use the optimal monetary policy rule from RANK model in the HANK model, 3) A rule without any endogenous monetary policy response, implying that the real rate is constant as in Section 3. Under the optimal monetary policy in HANK the aggregate welfare loss corresponds to -0.84% consumption-equivalents, which is a significant improvement compared to a passive monetary policy which features a welfare loss of -1.20%. Figure 18 shows that the bulk of the reduced welfare loss is accounted for by low-wealth households, whereas the households at the very top of the wealth distribution are actually hurt by moving from the constant real rate policy to the optimal policy.[33] Using the optimal RANK-policy – where rates increase more than under a constant-r policy, but less than under the optimal HANK-policy – reduces the welfare loss significantly compared to the constant-r policy, but less so than using the more hawkish policy obtained in the HANK model.

[32]See appendix C.7 for details.

[33]The welfare loss at the top of wealth distribution occurs because the labor wedge $u'(c_{it})\, e_{it} w_t - v'(L_t)$ (which is zero in the optimal allocation) gets more distorted for these households as monetary policy becomes more hawkish. This effect is limited at the bottom of the wealth distribution because it is dominated by the welfare effects from the borrowing constraint.

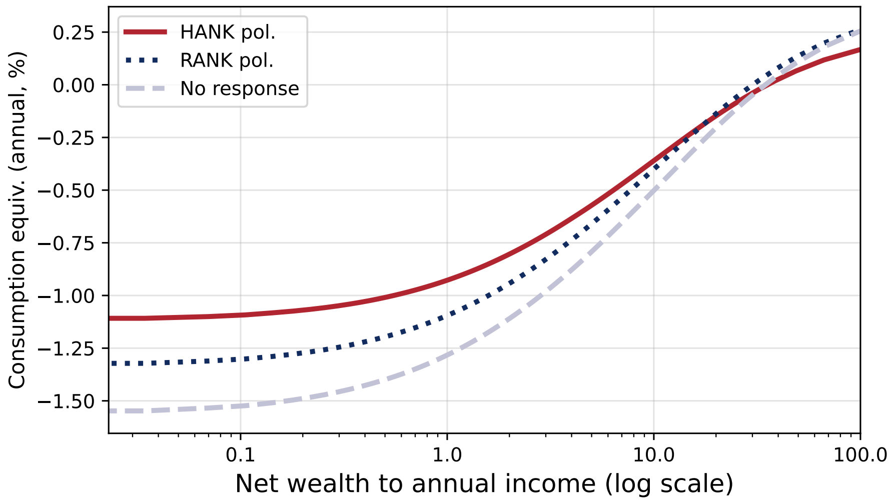


Figure 18: Welfare effects of three different policies in HANK

Note: Welfare effects under three different policies in the HANK model. "*HANK pol.*" corresponds to the optimal monetary policy in the baseline HANK model. "*RANK pol.*" computes the welfare loss in the HANK model using the optimal monetary policy obtained in the RANK model. "*No response*" corresponds to a constant real rate rule.

6 Conclusion

This paper has studied two questions: 1) What is the *positive* transmission of supply shocks in a small open economy with heterogeneous agents? 2) How is the *optimal* monetary policy response to such cost-push shocks shaped by heterogeneity and openness of the economy? Regarding question 1) I showed analytically that the propagation of inflationary supply shocks depends crucially on the incidence of profits and labor income as well as the degree of nominal price and wage rigidities.

For question 2) I showed the that the finding of a more accommodative optimal stabilization policy in HANK typically found in closed economy settings is fragile in the open economy as the open economy dimension has the potential to weaken the link between the policy rate and the demand for domestic goods. This allows the planner to exploit large appreciations of the domestic currency to alleviate inflationary pressures, thus stabilizing real income.

References

Acharya, Sushant, and Edouard Challe. 2024. "Inequality and Optimal Monetary Policy in the Open Economy". *Working Paper.*

Acharya, Sushant, Edouard Challe and Keshav Dogra. 2023. "Optimal monetary policy according to HANK". Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, *American Economic Review* 113 (7): 1741–1782.

Aiyagari, S. Rao. 1994. "Uninsured idiosyncratic risk and aggregate saving". Publisher: MIT Press, *The Quarterly Journal of Economics* 109 (3): 659–684.

———. 1995. "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting". *Journal of Political Economy* 103, no. 6 (December): 1158–1175.

Andersen, Asger Lau, Niels Johannesen and Adam Sheridan. 2024. "Dynamic spending responses to wealth shocks: Evidence from quasi-lotteries on the stock market". Publisher: CEPR Discussion Paper No. DP16338, *American Economic Review: Insights* Forthcoming.

Angeletos, George-Marios, Chen Lian and Christian K. Wolf. 2024. *Can Deficits Finance Themselves?* Technical report. National Bureau of Economic Research.

Ascari, Guido, Efrem Castelnuovo and Lorenza Rossi. 2011. "Calvo vs. Rotemberg in a trend inflation world: An empirical investigation". Publisher: Elsevier, *Journal of Economic Dynamics and Control* 35 (11): 1852–1867.

Auclert, Adrien. 2019. "Monetary policy and the redistribution channel". Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, *American Economic Review* 109 (6): 2333–2367.

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie and Ludwig Straub. 2021. "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models". *Econometrica* 89 (5): 2375–2408.

Auclert, Adrien, Hugo Monnery, Matthew Rognlie and Ludwig Straub. 2023a. *Managing an Energy Shock: Fiscal and Monetary Policy.* Technical report. Working Paper.

Auclert, Adrien, Rodolfo Rigato, Matthew Rognlie and Ludwig Straub. 2024a. "New Pricing Models, Same Old Phillips Curves?" Publisher: Oxford University Press, *The Quarterly Journal of Economics* 139 (1): 121–186.

Auclert, Adrien, and Matthew Rognlie. 2018. *Inequality and aggregate demand.* Technical report. National Bureau of Economic Research.

Auclert, Adrien, Matthew Rognlie, Martin Souchier and Ludwig Straub. 2024b. *Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel.* Technical report. National Bureau of Economic Research.

Auclert, Adrien, Matthew Rognlie and Ludwig Straub. 2020. *Micro jumps, macro humps: Monetary policy and business cycles in an estimated HANK model.* Technical report. National Bureau of Economic Research.

———. 2023b. "The Intertemporal Keynesian Cross",

Baqaee, David Rezza, and Emmanuel Farhi. 2024. "Networks, Barriers, and Trade". *Econometrica* 92 (2): 505–541.

Bayer, Christian, Ralph Lütticke, Lien Pham-Dao and Volker Tjaden. 2019. "Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk". Publisher: Wiley Online Library, *Econometrica* 87 (1): 255–290.

Benigno, Gianluca, and Pierpaolo Benigno. 2003. "Price stability in open economies". Publisher: Wiley-Blackwell, *The Review of Economic Studies* 70 (4): 743–764.

Bewley, Truman. 1986. "Stationary monetary equilibrium with a continuum of independently fluctuating consumers". Publisher: North-Holland Amsterdam, *Contributions to mathematical economics in honor of Gérard Debreu* 79.

Bhandari, Anmol, David Evans, Mikhail Golosov and Thomas Sargent. 2023. *Efficiency, Insurance, and Redistribution Effects of Government Policies.* Technical report. Working paper.

Bhandari, Anmol, David Evans, Mikhail Golosov and Thomas J. Sargent. 2021. "Inequality, Business Cycles, and Monetary-Fiscal Policy". Publisher: Wiley Online Library, *Econometrica* 89 (6): 2559–2599.

Bianchi, Francesco, Cosmin Ilut and Hikaru Saijo. 2023. "Diagnostic Business Cycles". Publisher: Oxford University Press US, *Review of Economic Studies,* rdad024.

Bilbiie, Florin O. 2020. "The new keynesian cross". Publisher: Elsevier, *Journal of Monetary Economics* 114:90–108.

———. 2024. "Monetary Policy and Heterogeneity: An Analytical FrameworkI". *The Review of Economic Studies,*

Blanchard, Olivier J., and Jordi Gali. 2007. *The Macroeconomic Effects of Oil Shocks: Why are the 2000s so different from the 1970s?*

Bobasu, Alina, Michael Dobrew and Amalia Repele. 2024. "Energy price shocks, monetary policy and inequality". *Available at SSRN 4775771.*

Bodenstein, Martin, Luca Guerrieri and Christopher J. Gust. 2013. "Oil shocks and the zero bound on nominal interest rates". Publisher: Elsevier, *Journal of International Money and Finance* 32:941–967.

Boehm, Christoph E., Aaron Flaaen and Nitya Pandalai-Nayar. 2019. "Input linkages and the transmission of shocks: Firm-level evidence from the 2011 Tōhoku earthquake". Publisher: MIT Press One Rogers Street, Cambridge, MA 02142-1209, USA journals-info … *Review of Economics and Statistics* 101 (1): 60–75.

Boehm, Christoph E., Andrei A. Levchenko and Nitya Pandalai-Nayar. 2023. "The long and short (run) of trade elasticities". Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, *American Economic Review* 113 (4): 861–905.

Bordalo, Pedro, Nicola Gennaioli and Andrei Shleifer. 2018. "Diagnostic Expectations and Credit Cycles". *The Journal of Finance* 73, no. 1 (February): 199–227.

Brunnermeier, Markus K., Sergio A. Correia, Stephan Luck, Emil Verner and Tom Zimmermann. 2023. *The debt-inflation channel of the German hyperinflation.* Technical report. National Bureau of Economic Research.

Burstein, Ariel, and Gita Gopinath. 2014. "International prices and exchange rates". In *Handbook of international economics,* 4:391–451. Elsevier.

Campa, Jose Manuel, and Linda S. Goldberg. 2005. "Exchange rate pass-through into import prices". Publisher: MIT Press 238 Main St., Suite 500, Cambridge, MA 02142-1046, USA journals … *Review of Economics and Statistics* 87 (4): 679–690.

Carroll, Christopher D. 2006. "The method of endogenous gridpoints for solving dynamic stochastic optimization problems". Publisher: Elsevier, *Economics letters* 91 (3): 312–320.

Chan, Jenny, Sebastian Diz and Derrick Kanngiesser. 2024. "Energy prices and household heterogeneity: Monetary policy in a gas-tank". Publisher: Elsevier, *Journal of Monetary Economics,* 103620.

Chari, Varadarajan V., and Patrick J. Kehoe. 1999. "Optimal fiscal and monetary policy". Publisher: Elsevier, *Handbook of macroeconomics* 1:1671–1745.

Chen, Sihao, Michael B. Devereux, Kang Shi and Juanyi Xu. 2023. "Consumption heterogeneity and monetary policy in an open economy". Publisher: Elsevier, *Journal of Monetary Economics* 140:1–15.

Chetty, Raj, Adam Guren, Day Manoli and Andrea Weber. 2011. "Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins". *American Economic Review* 101 (3): 471–75.

Chodorow-Reich, Gabriel, Plamen T. Nenov and Alp Simsek. 2021. "Stock market wealth and the real economy: A local labor market approach". Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, *American Economic Review* 111 (5): 1613–1657.

Christiano, Lawrence J., Mathias Trabandt and Karl Walentin. 2011. "Introducing financial frictions and unemployment into a small open economy model". Publisher: Elsevier, *Journal of Economic Dynamics and Control* 35 (12): 1999–2041.

Corsetti, Giancarlo, and Paolo Pesenti. 2001. "Welfare and macroeconomic interdependence". Publisher: MIT Press, *The Quarterly Journal of Economics* 116 (2): 421–445.

Cravino, Javier, Ting Lan and Andrei A. Levchenko. 2020. "Price stickiness along the income distribution and the effects of monetary policy". Publisher: Elsevier, *Journal of Monetary Economics* 110:19–32.

Davila, Eduardo, and Andreas Schaab. 2023. "Optimal Monetary Policy with Heterogeneous Agents: A Timeless Ramsey Approach". *Available at SSRN 4102028,*

Dávila, Eduardo, and Andreas Schaab. 2023. *Welfare assessments with heterogeneous individuals.* Technical report. National Bureau of Economic Research.

De Ferra, Sergio, Kurt Mitman and Federica Romei. 2020. "Household heterogeneity and the transmission of foreign shocks". Publisher: Elsevier, *Journal of International Economics* 124:103303.

Del Negro, Marco, Marc P. Giannoni and Frank Schorfheide. 2015. "Inflation in the great recession and new keynesian models". Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203-2425, *American Economic Journal: Macroeconomics* 7 (1): 168–196.

Diz, Sebastian, Mario Giarda and Damián Romero. 2023. "Inequality, nominal rigidities, and aggregate demand". Publisher: Elsevier, *European Economic Review* 158:104529.

Doepke, Matthias, and Martin Schneider. 2006. "Inflation and the redistribution of nominal wealth". Publisher: The University of Chicago Press, *Journal of Political Economy* 114 (6): 1069–1097.

Druedahl, Jeppe, Søren Hove Ravn, Laura Sunder-Plassmann, Jacob Marott Sundram and Nicolai Waldstrøm. 2022. "The Transmission of Foreign Demand Shocks".

Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin. 2000. "Optimal monetary policy with staggered wage and price contracts". Publisher: Elsevier, *Journal of monetary Economics* 46 (2): 281–313.

Faccini, Renato, Seungcheol Lee, Ralph Luetticke, Morten O. Ravn and Tobias Renkin. 2024. *Financial frictions: Micro vs. macro volatility.* Technical report. Danmarks Nationalbank Working Papers.

Fagereng, Andreas, Martin B. Holm and Gisle J. Natvik. 2021. "MPC heterogeneity and household balance sheets". *American Economic Journal: Macroeconomics* 13 (4): 1–54.

Faia, Ester, and Tommaso Monacelli. 2008. "Optimal Monetary Policy in a Small Open Economy with Home Bias". *Journal of Money, Credit and Banking* 40, no. 4 (June): 721–750.

Farhi, Emmanuel, and Iván Werning. 2014. "Dilemma Not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows". *IMF Economic Review* 62, no. 4 (November): 569–605.

———. 2019. "Monetary policy, bounded rationality, and incomplete markets". Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, *American Economic Review* 109 (11): 3887–3928.

Fischer, Stanley. 1976. "Wage-indexation and macro-economic stability". Publisher: Cambridge, Mass.: MIT Dept. of Economics,

Floden, Martin, and Jesper Lindé. 2001. "Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance?" Publisher: Elsevier, *Review of Economic dynamics* 4 (2): 406–437.

Fuster, Andreas, Greg Kaplan and Basit Zafar. 2021. "What would you do with $500? Spending responses to gains, losses, news, and loans". Publisher: Oxford University Press, *The Review of Economic Studies* 88 (4): 1760–1795.

Gabaix, Xavier. 2020. "A Behavioral New Keynesian Model". *American Economic Review* 110, no. 8 (August): 2271–2327.

Gali, Jordi, and Tommaso Monacelli. 2005. "Monetary policy and exchange rate volatility in a small open economy". Publisher: Wiley-Blackwell, *The Review of Economic Studies* 72 (3): 707–734.

Galí, Jordi, David López-Salido and Javier Vallés. 2004. *Rule-of-thumb consumers and the design of interest rate rules.*

Galí, Jordi, Frank Smets and Rafael Wouters. 2012. "Unemployment in an estimated New Keynesian model". Publisher: University of Chicago Press Chicago, IL, *NBER macroeconomics annual* 26 (1): 329–360.

González, Beatriz, Galo Nuño, Dominik Thaler and Silvia Albrizio. 2022. "Firm heterogeneity, capital misallocation and optimal monetary policy". Publisher: Banco de espana working paper.

———. 2024. "Firm heterogeneity, capital misallocation and optimal monetary policy". Publisher: ECB Working Paper,

Gornemann, Nils, Keith Kuester and Makoto Nakajima. 2021. *Doves for the rich, hawks for the poor? distributional consequences of systematic monetary policy.* Technical report. ECONtribute Discussion Paper.

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub and Iván Werning. 2021. "Monetary policy in times of structural reallocation". *University of Chicago, Becker Friedman Institute for Economics Working Paper*, nos. 2021-111.

Guvenen, Fatih, Greg Kaplan and Jae Song. 2014. "How risky are recessions for top earners?" Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, *American Economic Review* 104 (5): 148–153.

Guvenen, Fatih, Sam Schulhofer-Wohl, Jae Song and Motohiro Yogo. 2017. "Worker betas: Five facts about systematic earnings risk". Publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, *American Economic Review* 107 (5): 398–403.

Hagedorn, Marcus, Iourii Manovskii and Kurt Mitman. 2019. *The fiscal multiplier.* Technical report. National Bureau of Economic Research.

Huggett, Mark. 1993. "The risk-free rate in heterogeneous-agent incomplete-insurance economies". Publisher: Elsevier, *Journal of economic Dynamics and Control* 17 (5-6): 953–969.

Imrohoroğlu, Ayşe. 1989. "Cost of business cycles with indivisibilities and liquidity constraints". Publisher: The University of Chicago Press, *Journal of Political economy* 97 (6): 1364–1383.

Johnson, David S., Jonathan A. Parker and Nicholas S. Souleles. 2006. "Household expenditure and the income tax rebates of 2001". Publisher: American Economic Association, *American Economic Review* 96 (5): 1589–1610.

Kaplan, Greg, Benjamin Moll and Giovanni L. Violante. 2018. "Monetary policy according to HANK". *American Economic Review* 108 (3): 697–743.

Kose, M. Ayhan. 2002. "Explaining business cycles in small open economies:'How much do world prices matter?'" Publisher: Elsevier, *Journal of International Economics* 56 (2): 299–327.

Le Grand, François, Alaïs Martin-Baillon and Xavier Ragot. 2021. "Should monetary policy care about redistribution? Optimal fiscal and monetary policy with heterogeneous agents".

LeGrand, François, and Xavier Ragot. 2022. *Optimal Policies with Heterogeneous Agents: Truncation and Transitions.* Technical report. Working Paper, SciencesPo.

Lorenzoni, Guido, and Iván Werning. 2023. "Wage price spirals". *Forthcoming, Brookings Papers in Economic Activity,*

Luetticke, Ralph. 2021. "Transmission of monetary policy with heterogeneity in household portfolios". *American Economic Journal: Macroeconomics* 13 (2): 1–25.

Mankiw, N. Gregory, and Ricardo Reis. 2002. "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve". Publisher: MIT Press, *The Quarterly Journal of Economics* 117 (4): 1295–1328.

McKay, Alisdair, and Christian K. Wolf. 2023. "Optimal Policy Rules in HANK",

Mendoza, Enrique G. 1995. "The terms of trade, the real exchange rate, and economic fluctuations". Publisher: JSTOR, *International Economic Review,* 101–137.

Nakamura, Emi, and Jón Steinsson. 2008. "Five facts about prices: A reevaluation of menu cost models". Publisher: MIT Press, *The Quarterly Journal of Economics* 123 (4): 1415–1464.

Nuño, Galo, and Carlos Thomas. 2022. "Optimal redistributive inflation". Publisher: JSTOR, *Annals of Economics and Statistics,* no. 146, 3–64.

Obstfeld, Maurice, and Kenneth Rogoff. 2000. "New directions for stochastic open economy models". Publisher: Elsevier, *Journal of international economics* 50 (1): 117–153.

Oskolkov, Aleksei. 2023. "Exchange rate policy and heterogeneity in small open economies". Publisher: Elsevier, *Journal of International Economics* 142:103750.

Pieroni, Valerio. 2022. "Energy Shortages and Aggregate Demand: Output Loss and Unequal Burden from HANK". *European Economic Review.*

Rotemberg, Julio J. 1982. "Monopolistic price adjustment and aggregate output". Publisher: Wiley-Blackwell, *The Review of Economic Studies* 49 (4): 517–531.

Rotemberg, Julio J., and Michael Woodford. 1996. *Imperfect competition and the effects of energy price increases on economic activity.*

Sbordone, Argia M. 2006. "US wage and price dynamics: A limited information approach". *FRB of New York Staff Report,* no. 256.

Schmitt-Grohé, Stephanie, and Martin Uribe. 2005. "Optimal fiscal and monetary policy in a medium-scale macroeconomic model". Publisher: MIT Press, *NBER Macroeconomics Annual* 20:383–425.

———. 2006. *Comparing two variants of Calvo-type wage stickiness.*

Schnabel, Isabel. 2022. "Monetary policy in a cost-of-living crisis". *intervento al Panel su 'Fight Against Inflation'alla IV edizione del Foro La Toja, La Toja* 30.

Shapiro, Matthew D., and Joel Slemrod. 2003. "Consumer response to tax rebates". Publisher: American Economic Association, *American Economic Review* 93 (1): 381–396.

Smets, Frank, and Rafael Wouters. 2003. "An estimated dynamic stochastic general equilibrium model of the euro area". Publisher: Oxford University Press, *Journal of the European economic association* 1 (5): 1123–1175.

———. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach". Publisher: American Economic Association, *American economic review* 97 (3): 586–606.

Smirnov, Danila. 2022. "Optimal monetary policy in hank". *Unpublished manuscript, Universitat Pompeu Fabra,* Storesletten, Kjetil, Chris I. Telmer and Amir Yaron. 2004. "Cyclical Dynamics in Idiosyncratic Labor Market Risk". *Journal of Political Economy* 112, no. 3 (June): 695–717.

Woodford, Michael. 2003. *Interest and Prices.*

Yang, Yucheng. 2022. "Redistributive Inflation and Optimal Monetary Policy". *Available at SSRN 4275770.*

Young, Eric R. 2010. "Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm and non-stochastic simulations". Publisher: Elsevier, *Journal of Economic Dynamics and Control* 34 (1): 36–41.

Appendix

A Derivations and Proofs

A.1 Derivation of eq. (18)

Starting from goods market clearing condition (17) and linearizing around the zero inflation steady state gives:

$$dY_t = dC_{H,t} + dC^*_{H,t}$$

Substituting in for $dC^*_{H,t}$ from (16):

$$dY_t = dC_{H,t} - \eta^* \alpha^* \left(dP^*_{H,t} - dP^*_{F,t} \right)$$

Using the law of one price $dP^*_{H,t} + d\mathcal{E}_t = dP_{H,t}, dP_{F,t} = d\mathcal{E}_t + dP^*_{F,t}$ and the definition of the CPI $dP_t = \alpha dP_{F,t} + (1-\alpha)\, dP_{H,t}$ one gets:

$$\begin{aligned} dY_t &= dC_{H,t} - \frac{1}{(1-\alpha)} \eta^* \alpha^* \left(dP_t - d\mathcal{E}_t - dP^*_{F,t} \right) \\ \Leftrightarrow dY_t &= dC_{H,t} + \frac{1}{(1-\alpha)} \eta^* \alpha^* dQ_t \end{aligned}$$

where the second line uses the definition of the real exchange rate $Q_t = \frac{\mathcal{E}_t P^*_{F,t}}{P_t}$. Since the real UIP condition (15) implies $dQ_t = 0$ under a constant real rate policy we have:

$$dY_t = dC_{H,t}$$

Using that $dP_{H,t} - dP_t = -\frac{\alpha}{(1-\alpha)} dQ_t = 0$, as well as the fact that equations (37)-(38) reduce to $C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t$ when $\frac{P_{H,t}}{P_t}$ is constant I obtain:

$$dY_t = (1-\alpha)\, dC_t$$

which is exactly equation (18).

A.2 Proof of Proposition 1

First note that under a constant-r rule the consumption function $\mathcal{C}_t$ depends only on the aggregate sequence of labor income and profits, i.e.:

$$C_t = \mathcal{C}_t \left(\{ Z_s, \Pi_s \}_{s=0}^{\infty} \right)$$

or, written in sequence space: $d\boldsymbol{C} = \mathcal{C}\left(\{\boldsymbol{Z}, \boldsymbol{\Pi}\}\right)$. Linearizing and subbing this into (18) gives:

$$d\boldsymbol{Y} = (1-\alpha)\left[\boldsymbol{M}^Z d\boldsymbol{Z} + \boldsymbol{M}^\Pi d\boldsymbol{\Pi}\right] \tag{A.1}$$

To derive Proposition 1 we need to derive an expression for labor income $d\boldsymbol{Z}$. To do this we start from the definition of profits (13). Using $\nu \to 0$ and $\alpha_X = 0$ we have that:

$$\begin{aligned}
\Pi_t &= \frac{P_{H,t}}{P_t} Y_t - Z_t - \frac{\mathcal{E}_t P^*_{X,t}}{P_t} Y_t \left(1-\alpha_L\right) - \frac{\theta^P}{2} \pi^2_{H,t} Y_t \\
&= \left[\frac{P_{H,t}}{P_t} - P^*_{X,t}\left(1-\alpha_L\right) - \frac{\theta^P}{2}\pi^2_{H,t}\right] Y_t - Z_t
\end{aligned}$$

Linearizing gives:

$$\begin{aligned}
d\Pi_t = \left[1 - \left(1-\alpha_L\right)\right] dY_t + d\left(\frac{P_{H,t}}{P_t}\right) - dZ_t - \left(1-\alpha_L\right) dP^*_{X,t} \\
\Leftrightarrow d\Pi_t = \alpha_L dY_t - dZ_t - \left(1-\alpha_L\right) dP^*_{X,t} \\
\Leftrightarrow dZ_t = \alpha_L dY_t - d\Pi_t - \left(1-\alpha_L\right) dP^*_{X,t} \qquad \text{(A.2)}
\end{aligned}$$

Assume that we are only interested in a markup shock so $dP^*_{X,t} = 0$. Writing (A.2) in sequence-space and Substituting into (A.1) gives:

$$\begin{aligned}
d\boldsymbol{Y} &= (1-\alpha)\left[\boldsymbol{M}^Z\left[\alpha_L d\boldsymbol{Y} - d\boldsymbol{\Pi}\right] + \boldsymbol{M}^\Pi d\boldsymbol{\Pi}\right] \\
\frac{\boldsymbol{I}}{1-\alpha} d\boldsymbol{Y} &= \boldsymbol{M}^Z \alpha_L d\boldsymbol{Y} - \left[\boldsymbol{M}^Z - \boldsymbol{M}^\Pi\right] d\boldsymbol{\Pi}
\end{aligned}$$

which is eq. (19) in the main text.

A.3 Derivation of eq. (20)

Start by writing out eq. (19) at time t:

$$\frac{1}{1-\alpha} dY_t = \alpha_L \sum_{s=0}^{\infty} M^Z_{t,s} dY_s - \sum_{s=0}^{\infty}\left[M^Z_{t,s} - M^\Pi_{t,s}\right] d\Pi_s$$

Define $MPC_{t,s}$ as the time t MPC out of a lump-sum transfer at time s and $MPC_{i,t,s}$ the corresponding MPC for some households i in the population. Then, using the budget constraint (2) we have:

$$\frac{1}{1-\alpha} dY_t = \alpha_L \sum_{s=0}^{\infty} M^Z_{t,s} dY_s - \sum_{s=0}^{\infty}\left[\int MPC_{i,t,s} e_i d\mathcal{G} - \int MPC_{i,t,s} g'\left(a_i, \Pi_{ss}\right) d\mathcal{G}\right] d\Pi_s$$

Let us first rewrite the term $\int MPC_{i,t,s}e_i d\mathcal{G}$ using that:

$$\begin{aligned}
\mathbb{C}\text{ov}\left(MPC_{i,t,s}, e_i\right) &= \int \left(MPC_{i,t,s} - MPC_{t,s}\right)\left(e_i - \bar{e}\right) d\mathcal{G} \\
&= \int MPC_{i,t,s}e_i d\mathcal{G} - \int e_i MPC_{t,s} d\mathcal{G} - \int \bar{e} MPC_{i,t,s} d\mathcal{G} + \int \bar{e} MPC_{t,s} d\mathcal{G} \\
&= \int MPC_{i,t,s}e_i d\mathcal{G} - \bar{e} MPC_{t,s}
\end{aligned}$$

Implying that:

$$\int MPC_{i,t,s}e_i d\mathcal{G} = \bar{e} MPC_{t,s} + \mathbb{C}\text{ov}\left(MPC_{i,t,s}, e_i\right)$$

The derivation is the same for $\int MPC_{i,t,s}g'\left(a_i, \Pi_{ss}\right) d\mathcal{G}$ term, which gives:

$$\int MPC_{i,t,s}g'\left(a_i, \Pi_{ss}\right) d\mathcal{G} = \overline{g'\left(a_i, \Pi_{ss}\right)} MPC_{t,s} + \mathbb{C}\text{ov}\left(MPC_{i,t,s}, g'\left(a_i, \Pi_{ss}\right)\right)$$

Inserting we have:

$$\begin{aligned}
\sum_{s=0}^{\infty}\left[M_{t,s}^{Z} - M_{t,s}^{\Pi}\right] d\Pi_s &= \sum_{s=0}^{\infty}\left[\bar{e} MPC_{t,s} + \mathbb{C}\text{ov}\left(MPC_{i,t,s}, e_i\right)\right] d\Pi_s \\
&\quad - \sum_{s=0}^{\infty}\left[\overline{g'\left(a_i, \Pi_{ss}\right)} MPC_{t,s} + \mathbb{C}\text{ov}\left(MPC_{i,t,s}, g'\left(a_i, \Pi_{ss}\right)\right)\right] d\Pi_s \\
&= \sum_{s=0}^{\infty} MPC_{t,s}\left(\bar{e} - \overline{g'\left(a_i, \Pi_{ss}\right)}\right) d\Pi_s \\
&\quad + \sum_{s=0}^{\infty}\left[\mathbb{C}\text{ov}\left(MPC_{i,t,s}, e_i\right) - \mathbb{C}\text{ov}\left(MPC_{i,t,s}, g'\left(a_i, \Pi_{ss}\right)\right)\right] d\Pi_s
\end{aligned}$$

Since the Markov chain of e_i has mean 1 we have $\bar{e} = 1$. Similarly, if we assume $g\left(a_i, \Pi_{ss}\right)$ to linear in wealth we have $g\left(a_i, \Pi_{ss}\right) = \frac{a_i}{\int a_i d\mathcal{G}} \Pi_{ss}$ implying $\overline{g'\left(a_i, \Pi_{ss}\right)} = \frac{\int a_i d\mathcal{G}}{\int a_i d\mathcal{G}} = 1$. We are then left with:

$$\sum_{s=0}^{\infty}\left[M_{t,s}^{Z} - M_{t,s}^{\Pi}\right] d\Pi_s = \sum_{s=0}^{\infty}\left[\mathbb{C}\text{ov}\left(MPC_{i,t,s}, e_i\right) - \mathbb{C}\text{ov}\left(MPC_{i,t,s}, g'\left(a_i, \Pi_{ss}\right)\right)\right] d\Pi_s$$

Writing this in sequence space and subbing into eq. (19) yields eq. (20) in the main text:

$$\frac{\boldsymbol{I}}{1-\alpha} d\boldsymbol{Y} = \alpha_L \boldsymbol{M}^Z d\boldsymbol{Y} - \left[\mathbb{C}\text{ov}\left(\boldsymbol{M}^i, e_i\right) - \mathbb{C}\text{ov}\left(\boldsymbol{M}^i, g'\left(e_i, a_i\right)\right)\right] d\boldsymbol{\Pi}$$

A.4 Philips curve pass-through matrices

This section derives the expression for the pass-through matrices $\boldsymbol{\kappa}^P, \boldsymbol{\kappa}^W$ appearing in eq. (12) and (14). The baseline analysis assumes a price Philips-curve of the form:

$$\pi_t\left(1+\pi_t\right) = \kappa^P\left(mc_t - \frac{1}{\mu}\right) + \beta\pi_{t+1}\left(1+\pi_{t+1}\right) + \kappa^P\varepsilon^P$$

or, in linearized form around the zero inflation steady state:

$$d\pi_t = \kappa^P\left(dmc_t + d\varepsilon_t^P\right) + \beta d\pi_{t+1} \tag{A.3}$$

Imposing $d\pi_\infty = 0$ we have that $d\pi_t = \kappa^P\sum_{s=0}^{\infty}\beta^s\left(dmc_s + d\varepsilon_s^P\right)$. This equation can be written in sequence space as:

$$d\boldsymbol{\pi} = \kappa^P\boldsymbol{\Psi}\left(d\boldsymbol{mc} + d\boldsymbol{\varepsilon}^P\right) \tag{A.4}$$

where:

$$\boldsymbol{\Psi} = \begin{bmatrix} 1 & \beta & \beta^2 & \beta^3 & \cdots \\ 0 & 1 & \beta & \beta^2 & \cdots \\ 0 & 0 & 1 & \beta & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The final step is to rewrite the equation in terms of price levels instead of inflation. This yields:

$$d\boldsymbol{P} = \boldsymbol{\kappa}^P\left(d\boldsymbol{mc} + d\boldsymbol{\varepsilon}^P\right)$$

where $\boldsymbol{\kappa}^P = \kappa^P\boldsymbol{A}\boldsymbol{\Psi}$ and:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 1 & 1 & 0 & 0 & \cdots \\ 1 & 1 & 1 & 0 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

is a lower triangular matrix. $\boldsymbol{\kappa}^P$ has the following form:

$$\boldsymbol{\kappa}^P = \kappa^P \begin{bmatrix} 1 & \beta & \beta^2 & \beta^3 & \cdots \\ 1 & 1+\beta & \beta+\beta^2 & \beta^2+\beta^3 & \cdots \\ 1 & 1+\beta & 1+\beta+\beta^2 & \beta+\beta^2+\beta^3 & \cdots \\ 1 & 1+\beta & 1+\beta+\beta^2 & 1+\beta+\beta^2+\beta^3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Note that with a non-forward looking Philips curve, $\beta = 0$, then $\boldsymbol{\kappa}^P = \kappa^P \boldsymbol{A}$.
Regarding the New Keynesian wage Philips curve we have that the linearized NKWPC is:

$$d\pi_t^W = \kappa^W \left(\frac{1}{\phi} \frac{\varphi}{w} \mu^w L^{\frac{1}{\phi}-1} dL_t - \frac{\varphi}{w^2} \mu^w dw_t \right) + \beta d\pi_{t+1}^W$$

Using that in steady state $\varphi = \frac{w}{\mu^w}$ we get:

$$d\pi_t^W = \kappa^W \left(\frac{1}{\phi} dL_t - \frac{1}{W} dw_t \right) + \beta d\pi_{t+1}^W$$

Then, using the definition of $\boldsymbol{\Psi}$ above:

$$d\boldsymbol{\pi}^W = \kappa^W \boldsymbol{\Psi} \left(\frac{1}{\phi} d\boldsymbol{L} - \frac{1}{W} d\boldsymbol{w} \right)$$

To write in terms of wages in levels I use the defined $\boldsymbol{A}$ matrix above, noting that we have to pre-multiply with steady state wages (which was done implicitly for the NKPC as $P_H = 1$ in steady state):

$$\begin{aligned} d\boldsymbol{W} &= \kappa^W \boldsymbol{A} \boldsymbol{\Psi} \left(\frac{1}{\phi} W d\boldsymbol{L} - d\boldsymbol{w} \right) \\ &= \boldsymbol{\kappa}^W \left(\frac{1}{\phi} W d\boldsymbol{L} - d\boldsymbol{w} \right) \end{aligned}$$

with $\boldsymbol{\kappa}^W \equiv \kappa^W \boldsymbol{A} \boldsymbol{\Psi}$.

A.5 Proof of Proposition 2

The proof for Proposition 2 involves solving for $d\boldsymbol{\Pi}$ as a function of the markup shock and output. Starting from the equation:

$$d\Pi_t = \alpha_L dY_t - dZ_t - (1 - \alpha_L)\, dP_{X,t}^*$$

found in A.2 and using the assumption Leontiff production:

$$\begin{aligned} d\Pi_t &= \alpha_L dY_t - w dL_t - L dw_t - (1-\alpha_L)\, dP^*_{X,t} \\ &= \alpha_L (1-w)\, dY_t - \alpha_L dw_t - (1-\alpha_L)\, dP^*_{X,t} \end{aligned}$$

Then using the definition of real marginal costs $mc_t = \alpha_L w_t + (1-\alpha_L)\, P^*_{X,t}$ (which in steady state equals $\frac{1}{\mu}$):

$$d\Pi_t = \frac{\mu - 1}{\mu} dY_t - dmc_t$$

To proceed we go to the sequence-space notation and subtract the NKPC from the NKWPC using that $d\boldsymbol{P}_H = d\boldsymbol{P}$

$$\begin{aligned} d\boldsymbol{w} &= d\boldsymbol{W} - d\boldsymbol{P} \\ &= \boldsymbol{\kappa}^W (\phi d\boldsymbol{N} - d\boldsymbol{w}) - \boldsymbol{\kappa}^P (d\boldsymbol{mc} + d\boldsymbol{\mu}) \\ \Leftrightarrow \left(\boldsymbol{I} + \boldsymbol{\kappa}^W\right) d\boldsymbol{w} &= \boldsymbol{\kappa}^W \phi \alpha_L d\boldsymbol{Y} - \boldsymbol{\kappa}^P d\boldsymbol{mc} + \boldsymbol{\kappa}^P d\boldsymbol{\mu} \\ \Leftrightarrow \left(\boldsymbol{I} + \boldsymbol{\kappa}^W\right) d\boldsymbol{w} &= \boldsymbol{\kappa}^W \phi \alpha_L d\boldsymbol{Y} - \boldsymbol{\kappa}^P \left[\alpha_L d\boldsymbol{w} + (1-\alpha_L)\, d\boldsymbol{P}^*_{\boldsymbol{X}}\right] + \boldsymbol{\kappa}^P d\boldsymbol{\mu} \\ \Leftrightarrow \left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right) d\boldsymbol{w} &= \phi \alpha_L \boldsymbol{\kappa}^W d\boldsymbol{Y} - \boldsymbol{\kappa}^P (1-\alpha_L)\, d\boldsymbol{P}^*_{\boldsymbol{X}} + \boldsymbol{\kappa}^P d\boldsymbol{\mu} \end{aligned}$$

Using this yields the following expression for profits:

$$\begin{aligned} d\boldsymbol{\Pi} &= \frac{\mu - 1}{\mu} d\boldsymbol{Y} - d\boldsymbol{mc} \\ \Leftrightarrow d\Pi &= \left[\frac{\mu - 1}{\mu} - \alpha_L \left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \phi \alpha_L \boldsymbol{\kappa}^W\right] d\boldsymbol{Y} \\ &\quad + \alpha_L \left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \boldsymbol{\kappa}^P (1-\alpha_L)\, d\boldsymbol{P}^*_{\boldsymbol{X}} \\ &\quad - [(1-\alpha_L)]\, d\boldsymbol{P}^*_{\boldsymbol{X}} - \alpha_L \left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \boldsymbol{\kappa}^P d\boldsymbol{\mu} \\ \Leftrightarrow d\Pi &= \left[\frac{\mu - 1}{\mu} - \alpha_L \left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \phi \alpha_L \boldsymbol{\kappa}^W\right] d\boldsymbol{Y} \\ &\quad - (1-\alpha_L) \left[1 - \alpha_L \left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \boldsymbol{\kappa}^P\right] d\boldsymbol{P}^*_{\boldsymbol{X}} \\ &\quad - \alpha_L \left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \boldsymbol{\kappa}^P d\boldsymbol{\mu} \end{aligned} \tag{A.5}$$

Focusing on the markup shock ($d\boldsymbol{P}_X^* = 0$) and subbing into eq. (19):

$$\frac{\boldsymbol{I}}{1-\alpha} d\boldsymbol{Y} = \boldsymbol{M}^Z \alpha_L d\boldsymbol{Y} - \left[\boldsymbol{M}^Z - \boldsymbol{M}^\Pi\right] d\boldsymbol{\Pi}$$

$$\Leftrightarrow \left[\frac{\boldsymbol{I}}{(1-\alpha)\,\alpha_L} - \boldsymbol{M}^Z - \left[\boldsymbol{M}^Z - \boldsymbol{M}^\Pi\right]\left(\left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \phi \alpha_L \boldsymbol{\kappa}^W - \frac{1}{\alpha_L}\frac{\mu - 1}{\mu}\right)\right] d\boldsymbol{Y}$$
$$= -\left[\boldsymbol{M}^Z - \boldsymbol{M}^\Pi\right]\left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \boldsymbol{\kappa}^P d\boldsymbol{\mu}$$

Defining:

$$\mathcal{M} = \left[\frac{\boldsymbol{I}}{(1-\alpha)\,\alpha_L} - \boldsymbol{M}^Z - \left[\boldsymbol{M}^Z - \boldsymbol{M}^\Pi\right]\left(\left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \phi \alpha_L \boldsymbol{\kappa}^W - \frac{1}{\alpha_L}\frac{\mu - 1}{\mu}\right)\right]^{-1}$$

We get Proposition 2:

$$d\boldsymbol{Y} = -\mathcal{M}\left[\boldsymbol{M}^Z - \boldsymbol{M}^\Pi\right]\left(\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L \boldsymbol{\kappa}^P\right)^{-1} \boldsymbol{\kappa}^P d\boldsymbol{\mu}$$

A.6 Proof of Proposition 3

I first derive the derivative of $d\mathbf{Y}$ w.r.t κ^P. To this end I will use the equations:

$$\boldsymbol{\Theta}^\mu \equiv \left[\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L W \boldsymbol{\kappa}^P\right]^{-1} \alpha_L W \boldsymbol{\kappa}^P$$
$$\boldsymbol{\Theta}^L \equiv -\left[\boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L W \boldsymbol{\kappa}^P\right]^{-1} \frac{\alpha_L W}{\phi} \boldsymbol{\kappa}^W$$
$$\mathcal{M} \equiv \left[\frac{\mathbf{I}}{(1-\alpha)\,\alpha_L} - \mathbf{M}^Z - \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right]\left(\boldsymbol{\Theta}^L - \frac{1}{\alpha_L}\frac{\mu-1}{\mu}\right)\right]^{-1}$$
$$d\mathbf{Y} = -\mathcal{M}\left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] \times \boldsymbol{\Theta}^\mu \times d\boldsymbol{\mu}$$
$$\boldsymbol{\kappa}^P = \kappa^P \boldsymbol{A}\boldsymbol{\Psi}$$

The derivative I am interested in is given by:

$$\frac{\partial d\mathbf{Y}}{\partial \kappa^P} = -\left[\frac{\partial \mathcal{M}}{\partial \kappa^P} \times \boldsymbol{\Theta}^\mu + \mathcal{M} \times \frac{\partial \boldsymbol{\Theta}^\mu}{\partial \kappa^P}\right]\left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] \times d\boldsymbol{\mu}$$

To evaluate the sign it thus suffices to derive the signs of $\frac{\partial \mathcal{M}}{\partial \kappa^P}, \frac{\partial \boldsymbol{\Theta}^\mu}{\partial \kappa^P}$. I start with $\frac{\partial \boldsymbol{\Theta}^\mu}{\partial \kappa^P}$. For expositional ease, define the objects:

$$\boldsymbol{K} = \alpha_L W \boldsymbol{A}\boldsymbol{\Psi}, \quad \boldsymbol{R} = \frac{1}{\kappa^P}\boldsymbol{I} + \frac{1}{\kappa^P}\boldsymbol{\kappa}^W + \alpha_L W \boldsymbol{A}\boldsymbol{\Psi}$$

such that $\boldsymbol{\Theta}^{\mu} = \boldsymbol{R}^{-1}\boldsymbol{K}$. The derivative is then given by $\frac{\partial \boldsymbol{\Theta}^{\mu}}{\partial \kappa^P} = -\boldsymbol{R}^{-1}\frac{\partial \boldsymbol{R}}{\partial \kappa^P}\boldsymbol{R}^{-1}\boldsymbol{K}$. Since $\frac{\partial \boldsymbol{R}}{\partial \kappa^P} = -\frac{1}{(\kappa^P)^2}\left(\boldsymbol{I} + \boldsymbol{\kappa}^W\right)$ I get:

$$\frac{\partial \boldsymbol{\Theta}^{\mu}}{\partial \kappa^P} = \frac{1}{\left(\kappa^P\right)^2}\boldsymbol{R}^{-1}\left(\boldsymbol{I} + \boldsymbol{\kappa}^W\right)\boldsymbol{R}^{-1}\boldsymbol{K} \geq 0$$

Next I derive the sign of $\frac{\partial \mathcal{M}}{\partial \kappa^P}$. Define:

$$\boldsymbol{B} = \frac{\mathbf{I}}{(1-\alpha)\,\alpha_L} - \mathbf{M}^Z - \left[\mathbf{M}^Z - \mathbf{M}^{\Pi}\right]\left(\boldsymbol{\Theta}^L - \frac{1}{\alpha_L}\frac{\mu-1}{\mu}\right)$$

such that $\mathcal{M} \equiv \boldsymbol{B}^{-1}$. The derivative is $\frac{\partial \mathcal{M}}{\partial \kappa^P} = -\boldsymbol{B}^{-1}\frac{\partial \boldsymbol{B}}{\partial \kappa^P}\boldsymbol{B}^{-1}$, with $\frac{\partial \boldsymbol{B}}{\partial \kappa^P} = \left[\mathbf{M}^Z - \mathbf{M}^{\Pi}\right]\frac{\partial \boldsymbol{\Theta}^L}{\partial \kappa^P}$ and $\frac{\partial \boldsymbol{B}}{\partial \kappa^P} = \left[\mathbf{M}^Z - \mathbf{M}^{\Pi}\right]\frac{\partial \boldsymbol{\Theta}^L}{\partial \kappa^P}$. Using the same steps as above I obtain $\frac{\partial \boldsymbol{\Theta}^L}{\partial \kappa^P} = -\boldsymbol{R}^{-1}\alpha_L W \boldsymbol{R}^{-1}\boldsymbol{K} < 0$. Combining I obtain that $\frac{\partial \mathcal{M}}{\partial \kappa^P}$ is positive:

$$\frac{\partial \mathcal{M}}{\partial \kappa^P} = -\boldsymbol{B}^{-1}\left[\mathbf{M}^Z - \mathbf{M}^{\Pi}\right]\frac{\partial \boldsymbol{\Theta}^L}{\partial \kappa^P}\boldsymbol{B}^{-1} \geq 0$$

Proofing that $\frac{\partial d\mathbf{Y}}{\partial \kappa^P} \leq 0$ whenever $\mathbf{M}^Z \geq \mathbf{M}^{\Pi}$.
For the derivative w.r.t the κ^W, $\frac{\partial d\mathbf{Y}}{\partial \kappa^W}$ we have:

$$\frac{\partial d\mathbf{Y}}{\partial \kappa^W} = -\left[\frac{\partial \mathcal{M}}{\partial \kappa^W} \times \boldsymbol{\Theta}^{\mu} + \mathcal{M} \times \frac{\partial \boldsymbol{\Theta}^{\mu}}{\partial \kappa^W}\right]\left[\mathbf{M}^Z - \mathbf{M}^{\Pi}\right] \times d\boldsymbol{\mu}$$

The derivative $\frac{\partial \boldsymbol{\Theta}^{\mu}}{\partial \kappa^W}$ has sign:

$$\frac{\partial \boldsymbol{\Theta}^{\mu}}{\partial \kappa^W} = -\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\Psi}\boldsymbol{R}^{-1}\boldsymbol{K} < 0$$

where $\boldsymbol{R} = \boldsymbol{I} + \boldsymbol{\kappa}^W + \alpha_L W \boldsymbol{\kappa}^P, \boldsymbol{K} = \alpha_L W \boldsymbol{\kappa}^P$. For the derivative $\frac{\partial \mathcal{M}}{\partial \kappa^W}$ we have:

$$\frac{\partial \mathcal{M}}{\partial \kappa^W} = -\boldsymbol{B}^{-1}\frac{\partial \boldsymbol{B}}{\partial \kappa^W}\boldsymbol{B}^{-1} = \boldsymbol{B}^{-1}\left[\mathbf{M}^Z - \mathbf{M}^{\Pi}\right]\frac{\partial \boldsymbol{\Theta}^L}{\partial \kappa^W}\boldsymbol{B}^{-1} \leq 0$$

since $\frac{\partial \boldsymbol{\Theta}^L}{\partial \kappa^W} = \frac{1}{\left(\kappa^W\right)^2}\boldsymbol{R}^{-1}\left(\boldsymbol{I} + \alpha_L W \boldsymbol{\kappa}^P\right)\boldsymbol{R}^{-1}\boldsymbol{K} < 0$. Since both derivatives are negative, we have:

$$\frac{\partial d\mathbf{Y}}{\partial \kappa^W} \geq 0$$

whenever $\mathbf{M}^Z \geq \mathbf{M}^{\Pi}$

B Model extensions

B.1 Wage indexation

Proposition 3 highlights the importance of real wage dynamics for the transmission of the shock in models with distribution effects. A natural remedy to stabilize real wages in volatile inflation environments is to index nominal wages to CPI inflation (e.g. Fischer (1976)). Specifically, indexing wages to CPI inflation at rate ω modifies the Rotemberg adjustment cost on nominal wages to $\frac{\theta^W}{2}\left(\left(1+\pi_t^W\right)/\left(1+\pi_t\right)^{\omega}-1\right)^2$, see Ascari et al. (2011).[34] The implied wage Philips curve is then given by:

$$\pi_t^W = \omega\pi_t + \kappa^W\left(\nu'\left(L_t\right) - \frac{w_t}{\mu^W}\right) + \beta\left(\pi_{t+1}^W - \omega\pi_{t+1}\right)$$

With wage indexation present in the Philips curve the effects of a markup shock captured in Proposition 2 modifies to:

Proposition 6. *The general equilibrium response of output $d\mathbf{Y}$ to a markup shock is with wage indexation:*

$$d\mathbf{Y} = -\mathcal{M}\left[\mathbf{M}^Z - \mathbf{M}^{\Pi}\right] \times \boldsymbol{\Theta}^{\mu} \times d\boldsymbol{\mu}, \tag{A.6}$$

where the pass-through to markups $\boldsymbol{\Theta}^{\mu}$ is given by:

$$\boldsymbol{\Theta}^{\mu} = \left[\boldsymbol{I} + \boldsymbol{\kappa}^W + (1-\omega)\,\alpha_L W \boldsymbol{\kappa}^P\right]^{-1} \alpha_L W\left(1-\omega\right)\boldsymbol{\kappa}^P. \tag{A.7}$$

Proof: appendix B.2.

Inspecting (A.6)-(A.7) one finds that the presence of wage indexation modified the response of output in a manner similar to the degree of price stickiness κ^P. This implies that a sufficient degree of wage indexation is able to dampen the effect on aggregate demand from the markup shock (Proposition 3) by stabilizing the real wage since a higher ω fundamentally ties nominal wage growth to inflation. Figure A.1 illustrates this numerically. With fully indexed wages real wages are fully stabilized, and there is zero redistribution occurring in the economy, thus leading to full demand stabilization.

[34]For tractability I assume that wages are indexed to current inflation, but this is not central to my results.

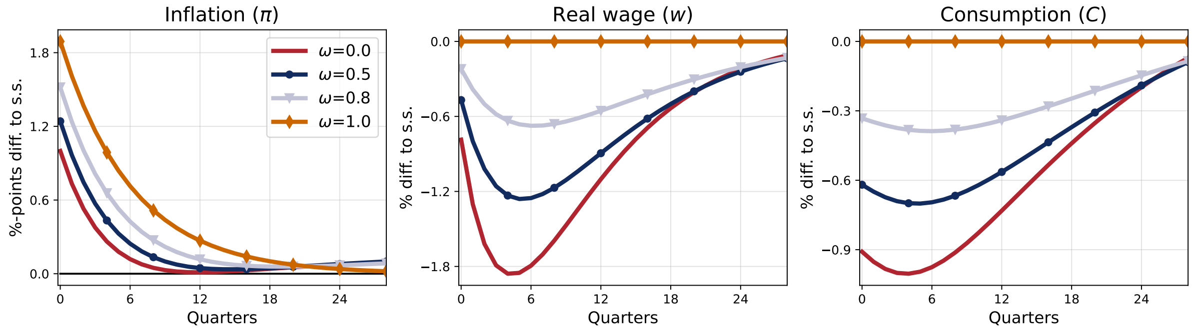


Figure A.1: Transmission of markup shock with varying degree of wage indexation

B.2 Proof of Proposition 6

The derivation of eq. (A.6) is unchanged with wage indexation, as only $\boldsymbol{\Theta}^{\mu}$ is affected. To derive $\boldsymbol{\Theta}^{\mu}$ under wage indexation, I start by linearizing the wage Philips curve with indexation:

$$\pi_t^W = \omega\pi_t + \kappa^W \left(\frac{\varphi L_t^{\frac{1}{\phi}}}{w_t} \mu^W - 1 \right) + \beta \left(\pi_{t+1}^W - \omega\pi_{t+1} \right)$$
$$\Rightarrow d\pi_t^W = \omega d\pi_t + \kappa^W \left(\frac{1}{\phi} dL_t - \frac{1}{W} dw_t \right) + \beta \left(d\pi_{t+1}^W - \omega d\pi_{t+1} \right)$$

Then, using the definition of $\boldsymbol{\Psi}$ above we get the sequence-space representation:

$$d\boldsymbol{\pi}^W - \omega d\boldsymbol{\pi} = \kappa^W \boldsymbol{\Psi} \left(\frac{1}{\phi} d\boldsymbol{L} - \frac{1}{W} d\boldsymbol{w} \right)$$

To write in levels instead of rates, I use $\boldsymbol{A}$ to get:

$$d\boldsymbol{W} - \omega W d\boldsymbol{P} = \boldsymbol{\kappa}^W \left(\frac{W}{\phi} d\boldsymbol{L} - d\boldsymbol{w} \right)$$

where $\boldsymbol{\kappa}^W \equiv \kappa^W \boldsymbol{A\Upsilon}$ as before.
To solve for $\boldsymbol{\Theta}^\mu$, I start by subbing the NKPC into the NWPC:

$$\begin{aligned}
d\boldsymbol{W} - \boldsymbol{\kappa}^W \left(\frac{W}{\phi} d\boldsymbol{L} - d\boldsymbol{w}\right) &= \omega W d\boldsymbol{P} \\
\Leftrightarrow d\boldsymbol{W} - W d\boldsymbol{P} + W d\boldsymbol{P} - \boldsymbol{\kappa}^W \left(\frac{W}{\phi} d\boldsymbol{L} - d\boldsymbol{w}\right) &= W\omega d\boldsymbol{P} \\
\Leftrightarrow d\boldsymbol{w} - \boldsymbol{\kappa}^W \left(\frac{W}{\phi} d\boldsymbol{L} - d\boldsymbol{w}\right) &= W(\omega - 1)\, d\boldsymbol{P} \\
\Leftrightarrow d\boldsymbol{w} - \boldsymbol{\kappa}^W \left(\frac{W}{\phi} d\boldsymbol{L} - d\boldsymbol{w}\right) &= W(\omega - 1)\, \boldsymbol{\kappa}^P \left(d\boldsymbol{mc} + d\boldsymbol{\varepsilon}^P\right) \\
\Leftrightarrow d\boldsymbol{w} - \boldsymbol{\kappa}^W \left(\frac{W}{\phi} d\boldsymbol{L} - d\boldsymbol{w}\right) &= W(\omega - 1)\, \boldsymbol{\kappa}^P \left(\alpha_L d\boldsymbol{w} + d\boldsymbol{\varepsilon}^P\right) \\
\Leftrightarrow d\boldsymbol{w} = -\left[\boldsymbol{I} + \boldsymbol{\kappa}^W + W(1-\omega)\, \boldsymbol{\kappa}^P \alpha_L\right]^{-1} & W(1-\omega)\, \boldsymbol{\kappa}^P d\boldsymbol{\varepsilon}^P
\end{aligned}$$

since $d\boldsymbol{mc} = \alpha_L d\boldsymbol{w}$ and $d\boldsymbol{\mu} = -\mu^2 d\boldsymbol{mc}$ we get the expression in eq. (A.7).

B.3 Wage-price spirals

A common argument against wage indexation is that it fuels additional inflation, which may induce a wage-price spiral (Lorenzoni and Werning 2023). This mechanism is at play in Figure A.1, where the increase in inflation is significantly larger with higher levels of wage indexation. Still the full-information rational expectations (FIRE) assumption underlying the model, rules out any explosive scenarios where current inflation is driven by increasing expectations of future inflation. I here consider an extension of the model where I allow for diagnostic expectations following Bianchi et al. (2023) in the two Philips curves. Diagnostic expectations are well suited to study my setting because they capture a behaviour where agents over-extrapolate expectations about future paths, which is exactly the main worry with surges in inflation. This type of expectation formation stands in contrast to other deviations from FIRE such as sticky expectations (Mankiw and Reis (2002)) or bounded rationality (Gabaix (2020)) where agents tend to underreact with respect to changes in aggregates.

Bordalo et al. (2018) introduced the Diagnostic Expectations (DE) model, where agents form expectations based on a prior distribution $\hat{h}$ of a shock dX, influenced by selective memory retrieval. The model incorporates the psychological principle that subjective probability assessments are biased towards events that are easily recalled. The DE model introduces a distorted density expressed through a subjective kernel, capturing the departure from full-information rational expectations (FIRE) only during the period of the shock. Building on this framework, Bianchi et al. (2023) extends the DE model by introducing long memory effects. The modified subjective kernel involves a product of lagged true distributions with memory weights, providing a more sophisticated representation of memory effects in expectations formation over time:

$$\hat{h}\left(dX_{t+1}\,|I_t\right) = h\left(dX_{t+1}\,|I_t\right)\left[\frac{h\left(dX_{t+1}\,|I_t\right)}{\prod_{j=1}^{J} h\left(dX_{t+1}\,|I_{t-j}\right)^{\alpha_j}}\right]^{\theta}\frac{1}{a}$$

Here, $h\left(dX_{t+1}\,|I_t\right)$ is the true density of dX_{t+1} given information I_t, $\hat{h}\left(dX_{t+1}\,|I_t\right)$ is the distorted density under DE, J represents the reference period, and the memory weights α_j are positive and sum to 1, and θ captures the strength of DE. In the main analysis I use the estimated values from Bianchi et al. (2023). This implementation implies that the expectation matrix $\boldsymbol{E}_t^{\boldsymbol{D}}$ is given by:

$$\boldsymbol{E}_t^{\boldsymbol{D}} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1+\theta\left(1-\sum_{j=1}^{t}\alpha_j\right) & 0 & \cdots & 0 \\ 0 & 0 & 1+\theta\left(1-\sum_{j=1}^{t}\alpha_j\right) & \cdots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \cdots & 1+\theta\left(1-\sum_{j=1}^{t}\alpha_j\right) \end{bmatrix}$$

With FIRE we have $\boldsymbol{E}_t^{\boldsymbol{D}}$ is a $T \times T$ identity matrix such that $\widetilde{\boldsymbol{\kappa}}_t^P = \boldsymbol{\kappa}_t^P, \widetilde{\boldsymbol{\kappa}}_t^W = \boldsymbol{\kappa}_t^W$.

Figure A.2 compares the responses to a markup shock in the baseline FIRE model with the impulse responses of model where diagnostic expectations are applied in the price Philips curve, the wage Philips curve, and both Philips curves respectively. Adding diagnostic expectations to the NKPC implies a larger initial inflation surge compared to FIRE because firms overestimate future inflation levels due to extrapolation, and therefore adjust prices more initially. This reduces the real wage and output more compared to the baseline. Adding diagnostic expectations to the NKWPC imply that nominal wages adjust more initially, thus stabilizing the real wage. In fact, under the specific calibration of diagnostic expectations, the overreaction in wage setting implies that the real wage increase and consumption/output initially increases. Finally, adding diagnostic expectations to both Philips curves simultaneously implies an even larger surge in inflation as the two Philips curves interact with each other - a wage-price spiral.

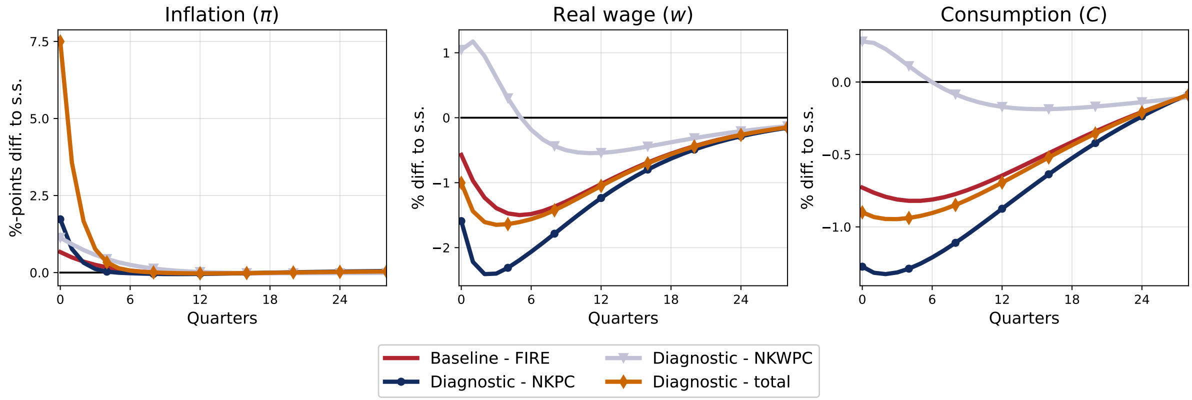


Figure A.2: Transmission of markup shock with diagnostic expectations

In order to evaluate if wage indexation is still a viable option in terms of real wage stabilization in the presence of a wage-price spiral it is worthwhile to inspect the analytical solution of the model. To start with, consider the following generalizations of the two Philips curves in eq. (21)-(22):

$$d\mathbf{P_H} = \widetilde{\boldsymbol{\kappa}}^P \left(d\mathbf{mc} + d\boldsymbol{\mu}\right) \tag{A.8}$$

$$d\mathbf{W} = \widetilde{\boldsymbol{\kappa}}^W \left(\phi d\mathbf{N} - d\mathbf{w}\right) \tag{A.9}$$

where $\widetilde{\boldsymbol{\kappa}}_t^P = \boldsymbol{E}_t^{\boldsymbol{D}} \boldsymbol{\kappa}_t^P, \widetilde{\boldsymbol{\kappa}}_t^W = \boldsymbol{E}_t^{\boldsymbol{D}} \boldsymbol{\kappa}_t^W$ and $\boldsymbol{E}_t^{\boldsymbol{D}}$ is the expectations matrix defined above. Under FIRE that matrix $\boldsymbol{E}_t^{\boldsymbol{D}}$ is a $T \times T$ identity matrix such that $\widetilde{\boldsymbol{\kappa}}_t^P = \boldsymbol{\kappa}_t^P, \widetilde{\boldsymbol{\kappa}}_t^W = \boldsymbol{\kappa}_t^W$, i.e. perfect foresight. With this notation in place I obtain the following solution under diagnostic expectations:

Proposition 7. *The general equilibrium response of output $d\mathbf{Y}$ to a markup shock under diagnostic expectations is:*

$$d\mathbf{Y} = -\mathcal{M}\left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] \times \widetilde{\boldsymbol{\Theta}}^\mu \times d\boldsymbol{\mu}, \tag{A.10}$$

where $\widetilde{\boldsymbol{\Theta}}^\mu$ determines the cyclicality of markups:

$$\widetilde{\boldsymbol{\Theta}}^\mu = \left[\boldsymbol{I} + \widetilde{\boldsymbol{\kappa}}^W + (1-\omega)\,\alpha_L W \widetilde{\boldsymbol{\kappa}}^P\right]^{-1} \alpha_L W (1-\omega)\, \widetilde{\boldsymbol{\kappa}}^P.$$

Proof: Follows from Proposition 6.

Note in particular that with diagnostic expectations we have $\operatorname{diag}\left(\widetilde{\boldsymbol{\kappa}}^P\right) \geq \operatorname{diag}\left(\boldsymbol{\kappa}^P\right)$ because agents tend to over-extrapolate based on current information, implying that these expectations manifest in the model as a price Philips curve with a larger slope.[35] As shown earlier this implies, all else equal, a larger effect of markup shocks on output.

[35]This would not be true with sticky expectations or bounded rationality.

Still, as shown in Figure A.3, wage indexation can successfully stabilize the real wage and output, but the concern about interaction effects between unanchored expectations and wage indexation seem to be well justified based on the excessive inflation response.

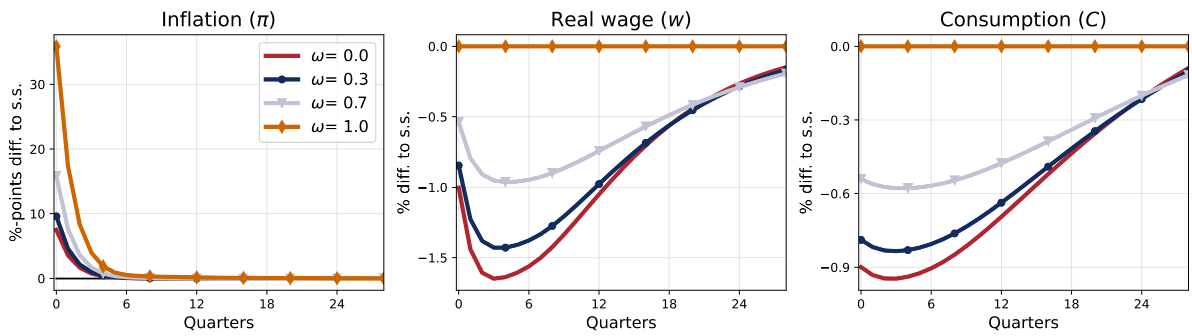


Figure A.3: Transmission of markup shock with varying degree of wage indexation in the diagnostic expectations model

B.4 Fisher effects

A mechanism often brought up in the context of large inflation surges is the redistribution that occurs between borrowers and savers when debt contracts are nominal (see e.g. Auclert 2019; Nuño and Thomas 2022; Brunnermeier et al. 2023). The baseline model assumes that households save in real assets, and thereby sidesteps this channel. To evaluate this effect I introduce a share θ^n of nominal bonds into the economy. The budget constraint of households is unchanged in real terms, but the total real return is then given by $r_t^a = \theta^n r_t^n + (1 - \theta^n) r_t$ where r_t^n is the real return on nominal bonds. I assume that these bonds are potentially long term and that they pay exponentially decaying coupons. A bond purchased at time 0 at nominal price q_t gives a stream $\{\delta^s\}_{s=0}^{\infty}$ of nominal payments. With $\delta = 0$ standard, short-term bonds are recovered. Arbitrage implies that the bond price follows:

$$q_t = \frac{1 + \delta q_{t+1}}{1 + i_t},$$

such that the expected real return on nominal bonds is the same as for real bonds. The return is given by:

$$1 + r_t^n = \frac{\frac{1+\delta q_t}{q_{t-1}}}{1 + \pi_t}.$$

With a constant-r rule as before the only effect on household behavior comes from an initial revaluation effect on nominal bonds which occurs at time 0 (only surprise inflation affects the return). The consumption function is now a function of real income, profits and the date 0 real return on assets, $C_t = \mathcal{C}\left\{Z_t, \Pi_t, r_0\right\}$. Linearizing in sequence-space and using $dr_0 = \theta^n dr_0^n$ we have that:

$$d\boldsymbol{C} = \boldsymbol{M}^Z d\boldsymbol{Z} + \boldsymbol{M}^\Pi d\Pi + \boldsymbol{M}^{r_0}\theta^n dr_0^n,$$

where $\boldsymbol{M}^{r_0}$ is a column vector whose entries reflect the response of consumption to the initial revaluation effect at time 0. Note that at the individual household level the entries in this vector would be positive for savers and negative for borrowers. To proceed, I linearize time 0 nominal returns to get:

$$dr_0^n = \underbrace{\frac{\delta}{q_{ss}} dq_0}_{\text{Long term bond effect}} - \underbrace{d\pi_0}_{\text{Fisher effect}}.$$

Thus the initial revaluation effect depends on two terms. With short bonds ($\delta = 0$) there is the usual Fisher effect of surprise inflation, whereby the value of nominal assets depreciates with the increase in inflation. With long term nominal bonds ($\delta > 0$) there is an additional effect since the bond has a duration of more than one period. For a given nominal interest rate, the increase in inflation also affects the value of the bond going forward, which is captured in the initial valuation dq_0. This effect can be found to be:

$$dq_0 = -\sum_{s=0}^{\infty} \frac{\delta^s}{\left(1+i_{ss}\right)^{2+s}} \left(1+\delta q_{ss}\right)\left(dr_{t+1+s} + d\pi_{t+1+s}\right)$$

High future real rates or inflation reduce the current price of the bond, with the effects being increasing in the longevity of the bonds δ. Using this expression, the overall effect on aggregate consumption is:

$$d\boldsymbol{C} = \underbrace{\boldsymbol{M}^Z d\boldsymbol{Z}}_{\text{Labor income}} + \underbrace{\boldsymbol{M}^\Pi d\Pi}_{\text{Profits}} \underbrace{- \boldsymbol{M}^{r_0}\theta^n \left[\sum_{s=0}^{\infty} \frac{\delta^s}{\left(1+i_{ss}\right)^{2+s}} \left(1+\delta q_{ss}\right) d\pi_{t+1+s} + d\pi_0\right]}_{\text{Fisher effects}}$$

As before a markup shock induces a negative effect on consumption from labor income and a positive effect from profits. If households hold nominal bonds, $\theta^n > 0$ there is an additional effect, which scales with the marginal propensity to spend out of time-0 unexpected returns $\boldsymbol{M}^{r_0}$. For savers this will tend to be a negative effect since the real value of their bonds get reduced by inflation, thus acting as a negative income shock. Borrowers experience the opposite, and will generally increase in consumption in response to this channel, leaving the overall sign of $\boldsymbol{M}^{r_0}$ uncertain. At the individual the MPC out of date-0 returns is the MPC out of a cash-transfer, times the initial net-nominal position of the household, $mpc_i \times a_i^n$. Aggregating we can write this as:

$$\boldsymbol{M}^{r_0} = \boldsymbol{M} \times A^n + \mathbb{C}\text{ov}\left(\boldsymbol{M}_i, a_i^n\right)$$

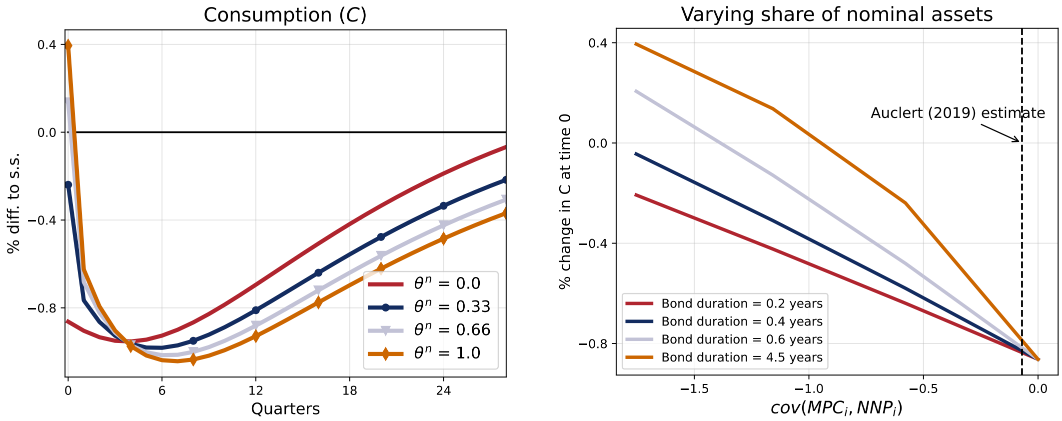
Given that the aggregate net-nominal position is generally positive, the first term is positive. The second term captures redistribution, between borrowers and savers, and is negative when borrowers have larger MPCs than savers. Auclert (2019) uses the Italian Survey of Household Income and Wealth to measure the covariance between MPCs and net nominal positions - i.e. $\mathbb{C}\text{ov}\left(\boldsymbol{M}_i, a_i^n\right)$ - and measure a positive, but quantitatively small effect of redistribution on consumption.

The left panel in Figure A.4 shows the effects of the markup shock on aggregate consumption for varying degree of nominal bonds, all with $\delta = 0.94$ to match an average duration of 4.5 years as in Doepke and Schneider (2006).[36] A larger share of nominal bonds imply redistribution towards borrowers, who in this model tend to have higher MPCs than savers. With enough nominal bonds this can potentially overturn the initial decline in consumption causing the aggregate response to turn positive. The response in the following quarters, however, is amplified relative to the baseline with real bonds because savers reduce their consumption, and though their initial time 0 MPCs are smaller than the corresponding MPCs of borrowers, their intertemporal MPCs are typically larger. Figure A.5 plots the intertemporal MPCs by borrowers and savers which clearly showcases this point.

However, the standard incomplete markets model has a tendency to overstate the aggregate effects of the Fisher redistribution channel. The right panel in Figure A.4 shows the change in consumption at time 0 as a function of the covariance $\mathbb{C}\text{ov}\left(\boldsymbol{M}_i, a_i^n\right)$, obtained by varying the share of nominal assets θ^n. With a larger share of nominal assets the model predicts a large negative covariance, implying strong effects of redistribution. However, the size of this covariance is many times larger than the empirical estimate from Auclert (2019).[37] If I calibrate θ^n to match the empirical covariance from Auclert (2019), the Fisher effect is quantitatively small, and the effect from declining real labor income dominates, and consumption declines following a markup shock, as in the baseline model.

[36] For this model exercise I set the borrowing constraint to minus one times the average quarterly steady-state labor income $\underline{a} = -Z_{ss}$ as in Kaplan et al. (2018).

[37] The model over-predicts the size of this covariance because all constraint households are up against the borrowing limit $\underline{a}$, whereas empirically there is typically bunching around the point $a = 0$, where the Fisher effect is limited. This can be remedied by introducing a borrowing premium, see Faccini et al. (2024).



(a) Response of aggregate consumption to a markup shock

(b) dC_0 with varying share of nominal bonds and duration

Figure A.4: Model with Fisher debt-inflation effects

Note: The left panel shows response of aggregate consumption for different shares of nominal assets θ^n fixing the bond duration to 4.5 years ($\delta = 0.95$). The right panel shows the response of aggregate consumption at time 0 as a function of the covariance between net-nominal positions and MPCs for different levels of bond durations.

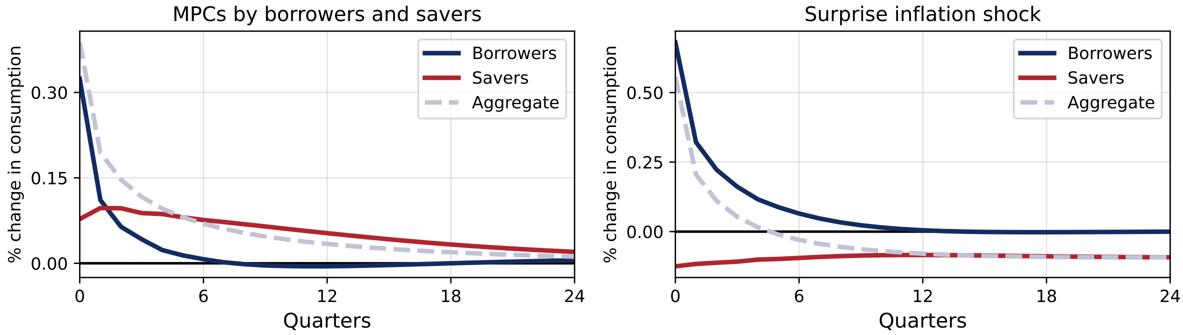


Figure A.5: Partial equilibrium responses by borrowers and savers

B.5 Neutrality of Cost-push shocks when $dP^*_{F,t} = dP^*_{X,t}$

Define $p_{X,t} = \frac{P_{X,t}}{P_t}$ as the price of imported materials in domestic CPI units. If $p_{X,t} = 0$ then the shock has no real effects on the domestic economy. To see when this case arises, consider the linearized versions of $p_{X,t}$ and Q_t under the law of one price:

$$dp_{X,t} = d\mathcal{E}_t + dP^*_{X,t} - dP_t$$
$$dQ_t = d\mathcal{E}_t + dP^*_{F,t} - dP_t$$

With a constant real rate and a UIP condition we have $dQ_t = 0$. Clearly if $dP^*_{X,t} = dP^*_{F,t}$ then $dp_{X,t} = 0$ and the shock to import prices have no effect on the domestic economy.

B.6 Cost-push shock

Subbing (A.2) in to (A.1) gives:

$$\begin{aligned} d\mathbf{Y} &= (1-\alpha)\left[\mathbf{M}^Z \alpha_L d\mathbf{Y} - \mathbf{M}^Z d\mathbf{\Pi} - \mathbf{M}^Z (1-\alpha_L)\, d\mathbf{P}_X^* + \mathbf{M}^\Pi d\mathbf{\Pi}\right] \\ \Leftrightarrow \frac{1}{1-\alpha} d\mathbf{Y} &= \alpha_L \mathbf{M}^Z d\mathbf{Y} - \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] d\mathbf{\Pi} - (1-\alpha_L)\, \mathbf{M}^Z d\mathbf{P}_\mathbf{X}^* \end{aligned}$$

which is eq. (26) in the main text. To derive (27), set $d\boldsymbol{\mu} = 0$ in (A.5), and insert:

$$\begin{aligned} \frac{1}{1-\alpha} d\mathbf{Y} = \alpha_L \mathbf{M}^Z d\mathbf{Y} &- \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right]\left[\frac{\mu-1}{\mu} - \alpha_L \boldsymbol{\Theta}^L\right] d\boldsymbol{Y} \\ &+ \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right](1-\alpha_L)\left[1-\boldsymbol{\Theta}^\mu\right] d\boldsymbol{P}_\mathbf{X}^* - (1-\alpha_L)\, \mathbf{M}^Z d\mathbf{P}_\mathbf{X}^* \end{aligned}$$

Solving for output one obtains:

$$d\mathbf{Y} = -\mathcal{M}\,(1-\alpha_L)\left\{\mathbf{M}^\Pi + \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right] \boldsymbol{\Theta}^\mu\right\} d\mathbf{P}_\mathbf{X}^*$$

where:

$$\mathcal{M} = \left[\frac{1}{1-\alpha} - \alpha_L \mathbf{M}^Z - \left[\mathbf{M}^Z - \mathbf{M}^\Pi\right]\left(\alpha_L \boldsymbol{\Theta}^L - \frac{\mu-1}{\mu}\mathbf{I}\right)\right]^{-1}$$

C Optimal Policy Appendix

C.1 Stylized model

The equations of the stylized model are laid out below. Eq. (A.11)-(A.12) describes how domestic consumption is split between domestic and foreign goods, eq. (A.13) describes the domestic CPI, eq. (A.14) defines the individual consumption function, eq. (A.15) defines aggregate consumption, eq. (A.16) defines the real wage, eq. (A.17) defines the production function of firms, eq. (A.18) is the real UIP condition, eq. (A.19) is the market clearing condition, eq. (A.20) defines exports as a function of the real exchange rate, and eq. (A.21) defines real labor income. Eq. (A.22) describes a static Philips-curve, with real marginal costs being defined in eq. (A.23) as a function $h(\bullet)$ of input prices and total production. For expositional simplicity firms are owned by foreigners, such that profits go abroad - the model can easily be extended to remedy this, but it would imply an extra aggregate state in the consumption function of households as in section 3.

$$C_{H,t} = (1-\alpha)\, C_t \tag{A.11}$$

$$C_{F,t} = \alpha C_t \tag{A.12}$$

$$P_t = \alpha P_{F,t} + (1-\alpha)\, P_{H,t} \tag{A.13}$$

$$c_{it} = c_i\,(Z_t, r_t) \tag{A.14}$$

$$C_t = \int c_{it}\,\mathrm{d}i \tag{A.15}$$

$$w_t = \frac{W_t}{P_t} \tag{A.16}$$

$$Y_t = f\,(L_t, X) \tag{A.17}$$

$$1 + r_t = \frac{1}{Q_t}\,(1 + r^*) \tag{A.18}$$

$$Y_t = C_{H,t} + C^*_{H,t} + \frac{\theta^P}{2}\pi^2_{H,t} Y_t \tag{A.19}$$

$$C^*_{H,t} = C^*_H\,(Q_t)\,, \quad C^{*\prime}_H\,(Q_t) > 0 \tag{A.20}$$

$$Z_t = w_t L_t \tag{A.21}$$

$$\pi_{H,t} = \kappa \left(mc_t - p_{H,t}\frac{1}{\mu} \right) \tag{A.22}$$

$$mc_t = h\,(w_t, P_{X,t}, Y_t) \tag{A.23}$$

$$P_{X,t} = P_X\left(Q_t P^*_{X,t}\right) \tag{A.24}$$

$$W_t = W \tag{A.25}$$

$$p_{F,t} = p_F\,(P^*_F Q_t) \tag{A.26}$$

$$P^*_{H,t} = \frac{p_{H,t}}{Q_t} \tag{A.27}$$

$$p_{F,t} = \frac{P_{F,t}}{P_t} \tag{A.28}$$

$$p_{H,t} = \frac{P_{H,t}}{P_t} \tag{A.29}$$

C.2 Derivation of delayed substitution model

I here present the model of delayed substitution. The setup closely follows Auclert et al. (2024b), but I derive the full non-linear solution. I focus on the problem of domestic households, but the problem for foreign households is isomorphic. Households maximize overall consumption C_t given by:

$$\left[\alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}}\, C_{H,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

subject to the constraint $C_t = C_{F,t}p_{F,t} + C_{H,t}p_{H,t}$, where $p_{F,t} = \frac{P_{F,t}}{P_t}$ and $p_{H,t} = \frac{P_{H,t}}{P_t}$ and P_t is the CPI price index. Optimization is subject to a Calvo-type friction. With probability θ^C households are not allowed to update the shares in their consumption bundle, whereas with probability $1-\theta^C$ they choose a new bundle.

Denote by $x_{H,t} = \frac{p_{H,t}C_{H,t}}{C_t}$ the share of consumption going to the home good, in units of the domestic CPI. A household that is allowed to update their consumption bundle in period t maximizes:

$$\max_{\mathring{x}_{H,t}} \sum_{s=0}^{\infty} \left(\beta\theta^C\right)^k \left[\alpha^{\frac{1}{\eta}} \left(\mathring{x}_{H,t} \frac{C_{t+s}}{p_{H,t+s}}\right)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} \left((1-\mathring{x}_{H,t}) \frac{C_{t+s}}{p_{F,t+s}}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where the notation $\mathring{x}_{H,t}$ expresses that this is targeted ratio for re-optimizing households. The first-order condition is:

$$\sum_{s=0}^{\infty} \left(\beta\theta^C\right)^s \left[(1-\alpha)^{\frac{1}{\eta}} \left(\frac{C_{t+s}}{p_{H,t+s}}\right)^{\frac{\eta-1}{\eta}} \mathring{x}_{H,t}^{\frac{-1}{\eta}} - \alpha^{\frac{1}{\eta}} \left(\frac{C_{t+s}}{p_{F,t+s}}\right)^{\frac{\eta-1}{\eta}} (1-\mathring{x}_{H,t})^{\frac{-1}{\eta}}\right] = 0$$

This may be rewritten as:

$$\left(\frac{\mathring{x}_{H,t}}{1-\mathring{x}_{H,t}}\right)^{-\frac{1}{\eta}} = \frac{\sum_{s=0}^{\infty} \left(\beta\theta^C\right)^s \alpha^{\frac{1}{\eta}} \left(\frac{C_{t+s}}{p_{F,t+s}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{s=0}^{\infty} \left(\beta\theta^C\right)^s (1-\alpha)^{\frac{1}{\eta}} \left(\frac{C_{t+s}}{p_{H,t+s}}\right)^{\frac{\eta-1}{\eta}}}$$

The solution for the target ratio is:

$$\mathring{x}_{H,t} = \frac{\left(\frac{A_t}{B_t}\right)^{-\eta}}{1 + \left(\frac{A_t}{B_t}\right)^{-\eta}}$$

where:

$$A_t = \sum_{s=0}^{\infty} \left(\beta\theta^C\right)^s \alpha^{\frac{1}{\eta}} \left(\frac{C_{t+s}}{p_{F,t+s}}\right)^{\frac{\eta-1}{\eta}}, \quad B_t = \sum_{s=0}^{\infty} \left(\beta\theta^C\right)^s (1-\alpha)^{\frac{1}{\eta}} \left(\frac{C_{t+s}}{p_{H,t+s}}\right)^{\frac{\eta-1}{\eta}}$$

Note that these can be written in recursive form as:

$$A_t = \alpha^{\frac{1}{\eta}} \left(\frac{C_t}{p_{F,t}}\right)^{\frac{\eta-1}{\eta}} + \beta\theta^C A_{t+1}, \quad B_t = (1-\alpha)^{\frac{1}{\eta}} \left(\frac{C_t}{p_{H,t}}\right)^{\frac{\eta-1}{\eta}} + \beta\theta^C B_{t+1}$$

Given the target ratio $\mathring{x}_{H,t}$ for households which may re-optimize, the aggregate ratio

$x_{H,t}$ follows the law of motion:

$$x_{H,t} = \left(1 - \theta^C\right) \mathring{x}_{H,t} + \theta^C x_{H,t-1}$$

C.3 Deriving eq. (32)

Before proceeding I define:

$$\partial_{X_t} \mathcal{W} \equiv \int E_0 u'\left(c_{it}\right) \partial_{X_t} c_{it} di$$

To derive eq. (32), I first use the fact that for two random variables x_i, y_i with density g_i, $\int x_i y_i g_i di = \mathbb{E}\, x_i\, \mathbb{E}\, y_i + \mathbb{C}\text{ov}\left(x_i, y_i\right)$ to get:

$$\partial_{X_t} \mathcal{W} = \int E_0 \left[\overline{u'\left(c_t\right)} \partial_{X_t} C_t + \mathbb{C}\text{ov}\left(u'\left(c_{it}\right), \partial_{X_t} c_{it}\right)\right] di$$

Then, utilizing a second order approximation of $\overline{u'\left(c_t\right)} = \int u'\left(c_{it}\right) di$ w.r.t c_{it} around C_t:

$$\begin{aligned}
\overline{u'\left(c_t\right)} &\approx \int u'\left(C_t\right) + u''\left(C_t\right)\left(c_{it} - C_t\right) + \frac{1}{2} u'''\left(C_t\right)\left(c_{it} - C_t\right)^2 di \\
&= u'\left(C_t\right) + \frac{1}{2} u'''\left(C_t\right) \mathbb{V}\text{ar}\left(c_{it}\right)
\end{aligned}$$

Further calculations give:

$$\begin{aligned}
&= u'\left(C_t\right) + \frac{1}{2} u'''\left(C_t\right) \mathbb{V}\text{ar}\left(c_{it}\right) = u'\left(C_t\right) + \frac{1}{2} \frac{u'''\left(C_t\right)}{u''\left(C_t\right)} \frac{C_t}{C_t} u''\left(C_t\right) \mathbb{V}\text{ar}\left(c_{it}\right) \\
&= u'\left(C_t\right) - \frac{1}{2} \vartheta \frac{1}{C_t} u''\left(C_t\right) \mathbb{V}\text{ar}\left(c_{it}\right) = u'\left(C_t\right) - \frac{1}{2} \vartheta \frac{u'\left(C_t\right)}{u'\left(C_t\right)} \frac{1}{C_t} u''\left(C_t\right) \mathbb{V}\text{ar}\left(c_{it}\right) \\
&= u'\left(C_t\right) + \frac{1}{2} \vartheta^P \vartheta^{CRRA} \frac{1}{u'\left(C_t\right)} \frac{1}{C_t^2} \mathbb{V}\text{ar}\left(c_{it}\right) = u'\left(C_t\right) + \frac{1}{C_t} \mathbb{V}\text{ar}\left(c_{it}\right)
\end{aligned}$$

where I use that for log-preferences, prudence equals $\vartheta^P = 2$ and the coefficient of relative risk aversion equals $\vartheta^{CRRA} = 1$. Combining this I obtain:

$$\partial_{X_t} \mathcal{W} = \int E_0 \left[\partial_{X_t} C_t \left(u'\left(C_t\right) + \frac{1}{C_t} \mathbb{V}\text{ar}\left(c_{it}\right)\right) + \mathbb{C}\text{ov}\left(u'\left(c_{it}\right), \partial_{X_t} c_{it}\right)\right] di$$

Setting $X_t = Z_t$ yields the equation in the main text.

C.4 Optimal policy under commitment

To resolve the time-inconsistency problem I follow Woodford (2003) and implement the optimal monetary policy under commitment. To this end I prepend the residuals in $\mathcal{H}(\boldsymbol{Z}, \boldsymbol{X}, \epsilon)$ with their period -1 (i.e. in the period before the MIT shock) values. Denoting by m_{-1} the pre-shock values of Lagrange multipliers the Lagrangian is:

$$\mathcal{L}(\boldsymbol{M}, \boldsymbol{Z}, \boldsymbol{X}) = \mathbb{E}_0 W + \begin{bmatrix} m_{-1} & \boldsymbol{M} \end{bmatrix}' \begin{bmatrix} h_{-1} & \mathcal{H} \end{bmatrix}$$

The FOC w.r.t the instruments is:

$$\nabla_{\boldsymbol{Z}} \mathbb{E}_0 \mathcal{W} + \begin{bmatrix} m_{-1} & \boldsymbol{M} \end{bmatrix}' \begin{bmatrix} \nabla_{\boldsymbol{Z}} h_{-1} & \nabla_{\boldsymbol{Z}} \mathcal{H} \end{bmatrix} = \boldsymbol{0}$$

The optimal policy under commitment is obtained when the initial values of the Lagrange multipliers m_{-1} are fixed at their steady state values. Discretion obtains when $m_{-1} = 0$. Note that if there are no forward looking constraints in $h_{-1}, \mathcal{H}$ then $\nabla_{\boldsymbol{Z}} h_{-1} = 0$ and one obtains time-consistent policy independently of the initial values of Lagrange multipliers. The previous setting assumes that $\mathcal{H}$ contains all model equations as residuals and the dimensionality of the system is therefore too large to handle numerically due to the infinite dimensionality of the households block.[38] I reduce the dimensionality by writing the system in a DAG-form following Auclert et al. (2021). In my case this effectively reduces the size of $\mathcal{H}$ to a dynamic system with 4 residuals and 4 unknowns such that the vector $\mathcal{H}$ is of size $T \cdot 4$ where T is the length of the transition path. Given that this removes a large number of forward-looking equations from $\mathcal{H}$ (and therefore their associated Lagrange multipliers) how does one obtain commitment in this case? The key is to include in the Lagrangian lagged aggregate utility $\mathcal{U}_{-1}$:

$$\mathcal{L}(\boldsymbol{M}, \boldsymbol{Z}, \boldsymbol{X}) = \mathcal{U}_{-1} + \mathbb{E}_0 \mathcal{W} + \begin{bmatrix} m_{-1} & \boldsymbol{M} \end{bmatrix}' \begin{bmatrix} h_{-1} & \mathcal{H} \end{bmatrix}$$

where:

$$\mathcal{U}_{-1} = \beta^{-1} \int \left[u\left(c_{-1}(e, a)\right) - v\left(L_{-1}\right) \right] d\mathcal{G}_{-1}(e, a)$$

[38] This is also the case after discretization.

In this form the Ramsey problem can be solved as follows:[39] First denote by $\mathcal{R}$ the non-linear equation system composed of the first-order condition of the planner:

$$\mathcal{R}\left(\boldsymbol{M},\boldsymbol{Z},\boldsymbol{X},\boldsymbol{\epsilon}\right)=\begin{bmatrix}\nabla_{\boldsymbol{Z}}\mathcal{W}+\nabla_{\boldsymbol{Z}}\boldsymbol{M}'\mathcal{H}\left(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{\epsilon}\right)\\ \nabla_{\boldsymbol{X}}\mathcal{W}+\nabla_{\boldsymbol{X}}\boldsymbol{M}'\mathcal{H}\left(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{\epsilon}\right)\\ \mathcal{H}\left(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{\epsilon}\right)\end{bmatrix}=\mathbf{0}$$

Under commitment it holds that $\mathcal{R}\left(\boldsymbol{M}_{ss},\boldsymbol{Z}_{ss},\boldsymbol{X}_{ss},\boldsymbol{\epsilon}_{ss}\right)=\mathbf{0}$. Collecting endogenous variables in a vector $\boldsymbol{R}=\left(\boldsymbol{M},\boldsymbol{Z},\boldsymbol{X}\right)$, and perturbing $\mathcal{R}$ w.r.t $\boldsymbol{R},\boldsymbol{\epsilon}$ around the steady state yields:

$$\begin{aligned}\mathcal{R}_{\boldsymbol{R}}d\boldsymbol{R}+\mathcal{R}_{\boldsymbol{\epsilon}}d\boldsymbol{\epsilon}&=\mathbf{0}\\ \Leftrightarrow d\boldsymbol{R}&=-\mathcal{R}_{\boldsymbol{R}}^{-1}\mathcal{R}_{\boldsymbol{\epsilon}}d\boldsymbol{\epsilon}\end{aligned}$$

where $\mathcal{R}_{\boldsymbol{R}},\mathcal{R}_{\boldsymbol{\epsilon}}$ are the Jacobians of $\mathcal{R}$ w.r.t $\boldsymbol{R},\boldsymbol{\epsilon}$.

C.5 Foreign currency denominated debt

As in the baseline model household can hold either domestic bonds or foreign bonds. The total level of assets of households i is given by $a_{it}=b_{it}+b_{it}^{*}$. Denote by ω^{B} the share of total assets invested in domestic bonds in the initial steady state. Then:

$$\begin{aligned}c_{it}+b_{it}+b_{it}^{*}&=\left(1+r_{t}^{b^{*}}\right)b_{it-1}^{*}+\left(1+r_{t}^{b}\right)b_{it-1}+Z_{t}e_{i}+g\left(e_{i}\right)\Pi_{t}-\tau\left(e_{i}\right),\\ \Leftrightarrow c_{it}+a_{it}&=\left(1+r_{t}^{b}\right)a_{it-1}+x_{it}+Z_{t}e_{i}+g\left(e_{i}\right)\Pi_{t}-\tau\left(e_{i}\right),\end{aligned}$$

with $x_{it}=\left[\left(1+r_{t}^{b^{*}}\right)-\left(1+r_{t}^{b}\right)\right]b_{it-1}^{*}$. I consider two scenarios: 1) The portfolio share ω is constant across individuals, and leverage ratios are therefore the same across all households, 2) portfolio shares vary across households ω_{i}, with poorer households being more leveraged in foreign bonds. For the first scenario I follow De Ferra et al. (2020) and set the steady state supply of foreign credit to 25% of total net wealth, $\frac{\int b_{i}^{*}di}{\int b_{i}^{*}+b_{i}di}=\frac{B^{*}}{A}=-0.25$. Since I assume the leverage ratio to be constant we have $b_{i}^{*}=\left(1-\omega\right)a_{i}$. The calibration then implies $\omega=1.25$. For the second scenario I assume that poorer households are more leveraged using the function form $b_{it-1}^{*}=\left(1-\omega\right)\left(k+a_{it-1}\right)$. I again follow De Ferra et al. (2020) and calibrate k,ω to jointly match 1) $\frac{B^{*}}{A}=-0.25$ (as before) and 2) The average gross debt of households with zero net wealth is 24% of average household yearly labor income. This yields $k=3.4,\omega=1.15$. In Figure A.6 I plot the debt-to-income ratio as a function of net wealth in the two calibrations.

[39] See Davila and Schaab (2023) for more details.

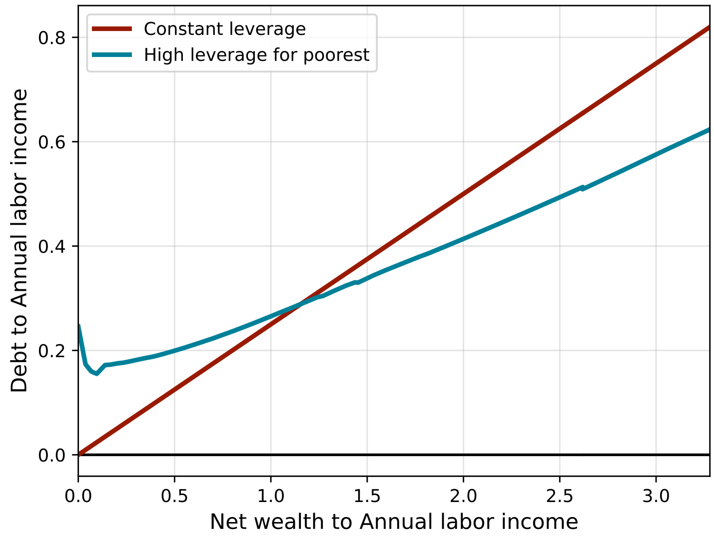


Figure A.6: Distribution of leverage in $NFA < 0$ HANK model

The currency valuation channel only affects the balance sheets of households in the initial period following a MIT shock since expected returns across foreign and domestic bonds are equalized going forward. The return on domestic bonds is the rate set by the domestic central bank, $r_0^b = r_{ss}$, whereas the return on foreign bonds (once converted to domestic currency) is:

$$r_0^{b^*} = (1 + r^*) \frac{Q_0}{Q_{ss}} - 1$$

C.6 Cyclical inequality

I introduce cyclical risk following Auclert and Rognlie (2018) and Acharya et al. (2023). The labor income received by individual i is:

$$z_{it} = Z_t \frac{e_i^{1+\xi \ln \frac{Z_t}{Z}}}{\int e_i^{1+\xi \ln \frac{Z_t}{Z}} di}$$

where the parameter ξ determines the cyclicality of cross-sectional income dispersion. $\xi = 0$ recovers the standard model with acyclical earnings dispersion whereas $\xi < 0$ corresponds to countercyclical income risk. For the calibration of ξ I follow Acharya et al. (2023) who uses the estimated values of cross-sectional idiosyncratic risk in peaks and troughs in the US from Storesletten et al. (2004). They estimate that the annual standard deviation increases from 0.12 to 0.21 when moving between the two states. I convert these two quarterly standard deviations by simulating an AR(1) process with persistence 0.95 (the estimated value in Storesletten et al. (2004)) at the quarterly level, calibrating the standard deviation of innovations to match the annual standard deviations of 0.12 to 0.21 respectively. This gives quarterly values of 0.124 and 0.217. Acharya et al. (2023) assume that GDP declines by 3% when moving from peak to trough. I approximate the decline in aggregate activity with aggregate real labor income, implying $\frac{d\sigma_z}{d\ln Z_t} = \frac{0.124-0.217}{0.03} = -3.1$ empirically. In the model I have:

$$\sigma_{z,t} = \text{sd}(\ln z_{it}) = \left(1 + \xi \ln \frac{Z_t}{Z}\right) \text{sd}(\ln e_i)$$

Implying:

$$\frac{d\sigma_{z,t}}{d\ln Z_t} = \xi \sigma_e$$

In the baseline calibration the cross-sectional standard deviation of log earnings σ_e is 0.5. Thus $\xi = \frac{-3.1}{0.5} = -6.2$ which is the value used in the experiment.

C.7 Welfare changes measured in consumption-equivalents

I compute consumption equivalent variation as follows. Let $\mathcal{P}_t$ denote the time t transition matrix which governs how households move across states (e, a). Note that this matrix contains both an exogenous component which governs transition across idiosyncratic earnings state (e), and an endogenous component which governs how households moves in the asset state space (a).

Collecting states in a tuple $s = (e, a)$, ex-ante expected lifetime utility of an individual with states e, a is:

$$\begin{aligned} V(s) &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t(s)\right) - \nu\left(L_t\right)\right] \\ &= \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \mathcal{P}_t\left(s^t \,|s\right) \left[u\left(c_t\left(s^t\right)\right) - \nu\left(L_t\right)\right] \end{aligned}$$

Let $\overline{V}(s)$ denote expected life-time utility in the absence of aggregate shocks. The consumption-equivalent variation $x(s)$ needed to make an individual with initial state s indifferent between the steady state, and a world where the aggregate shock materialises is defined by:

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \mathcal{P}_t\left(s^t \,|s\right) \left[u\left(c_t\left(s^t\right) + x(s)\right) - \nu\left(L_t\right)\right] = \overline{V}(s)$$

where I use the additive formulation for the consumption-equivalent following Dávila and Schaab (2023) since this allows me to compute an aggregate consumption-equivalent.